Discrete Model of TCP Congestion Control Algorithm with Round Dependent Loss Rate

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Motivation and Problem Statement

- Internet needs a tool to control its performance and resource sharing
- This service is provided at the end-to-end level by Transmission Control Protocol (TCP)
- TCP performs distributed flow control. It controls **performance** of the network to prevents it from **congestion** collapse.

Key **metrics** of congestion control performance:

- Average congestion window size
- Variance of of congestion window size

Motivation and Problem Statement

Application areas:

- Performance Evaluation and Capacity Planning of networking links and networks
- Throughput evaluation across end-to-end path
- Performance prediction for jitters sensitive applications (variance), e.g. media streams
- Choose cloud provider for fetching contents
- Support network administration and design
- Provides foundation for further research and development on transport layer protocols.

Distributed Flow Control

- Sliding window is amount of data which sender is allowed to inject in the network without acknowledgment
- Flow control means control on sliding window size. TCP uses set of algorithms to control its window size W

Additive Increase Multiplicative Decrease Algorithm (AIMD)

$$\begin{array}{lcl} W & \xrightarrow{\text{delivery}} & W+1 \\ \downarrow \text{loss} & & \\ W\alpha & & \end{array}$$

$\alpha < 1$ is rational number $\alpha = \dfrac{n}{m}, \quad n < m$

Details on Sliding Window Size

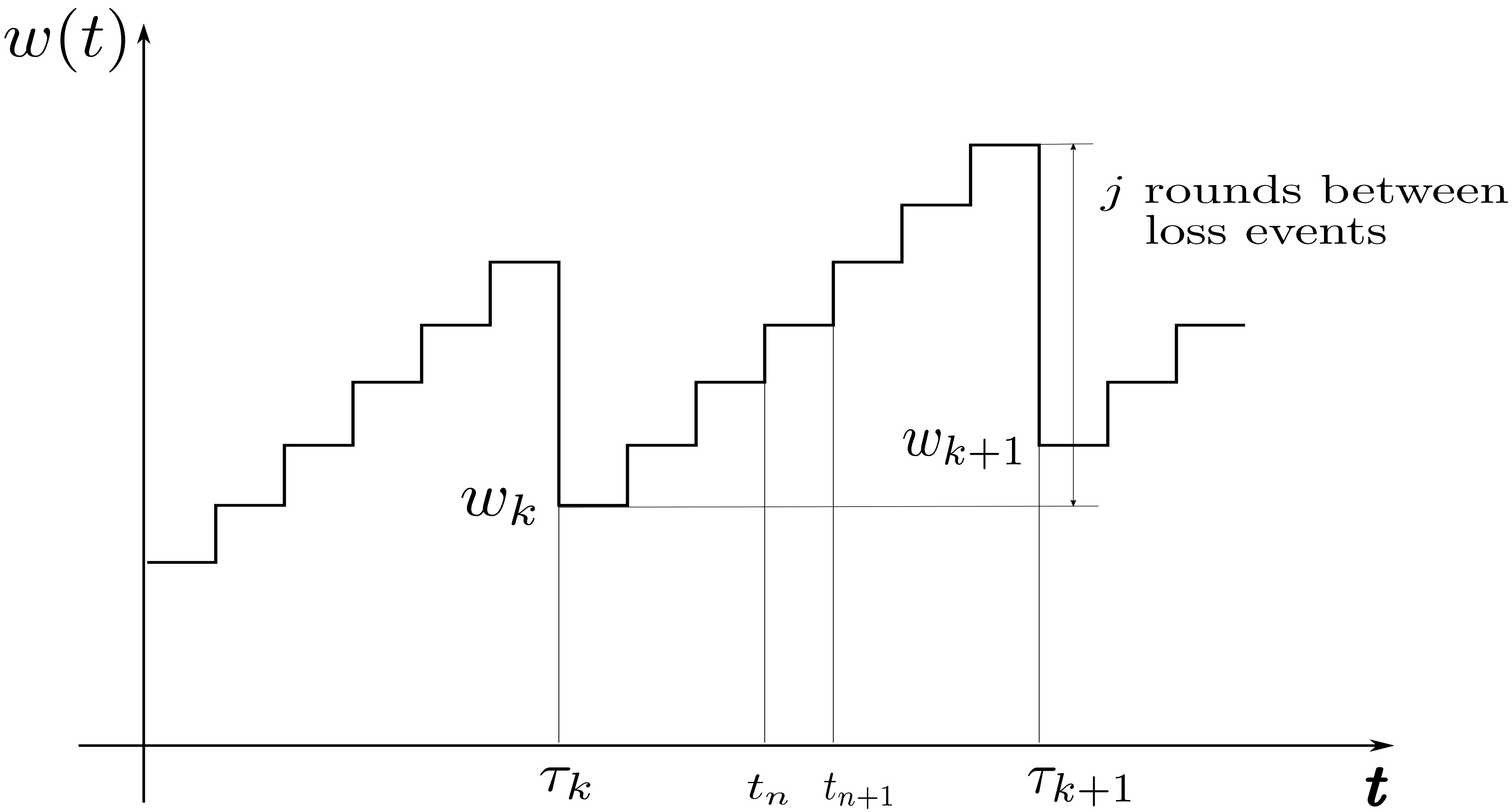

Figure 1: Evolution of TCP congestion window size

Main Definitions

Let $w(t)$ be cwnd size, t_n are AIMD-rounds end-points.

Let τ_k be equal to the first moment t_n, arrived after kth data loss event, i.e.

$$\tau_k = t_n : \; w(t_n + 0) = \left\lfloor \frac{w(t_n)n}{m} \right\rfloor, \; k = 1, 2. \ldots$$

and $\tau_{k_1} < \tau_{k_2}$, if $k_1 < k_2$.

Discrete distribution $\{f_j\}_{j \geq 0}$, i. e. the distribution of the number of the consequential rounds at the end of which $w(t)$ size increased.

In other words f_j is the probability that a sequence of j rounds without losses took place after a segment loss event occurred.

Ergodic Properties

According to the assumptions and definitions above the sequence $\{w_k = w(\tau_k + 0)\}_{k>0}$ is the **Markov chain**.

Let's define the expectation determined by f_j as $R = \sum_{j=1}^{\infty} j f_j$.

Theorem 1 *If R is finite then the Markov chain $\{w_k\}$ has steady state distribution.*

The Expectation Estimate

$$w_{k+1} = \left\lfloor \frac{(w_k + r_k)n}{m} \right\rfloor, \tag{1}$$

where r_k is the random variable with the distribution $\{f_j\}$.

$$w_{k+1} = \frac{(w_k + r_k)n - i_k}{m}, \tag{2}$$

where i_k is the integer random variable and $0 \leq i_k < m$. Difference equation (2) has a stationary solution

$$w_k^* = \sum_{j=0}^{\infty} \left(\frac{n}{m}\right)^{j+1} r_{k-j} - \frac{1}{m} \sum_{j=0}^{\infty} \left(\frac{n}{m}\right)^{j} i_{k-j}. \tag{3}$$

The Expectation Estimate

Theorem 2 *If the inequality* $R > \dfrac{m}{n}$ *holds then the stationary expectation of the congestion window size can be estimated as follows*

$$\mathsf{E}[w_k^*] \geq R\frac{n}{m-n} - \frac{m-1}{m-n}. \tag{4}$$

P R O O F. Processing of the expression (3) yields the following

$$\mathsf{E}[w_k^*] = \sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j+1}\mathsf{E}[r_{k-j}] - \frac{1}{m}\sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j}\mathsf{E}[i_{k-j}]. \tag{5}$$

By definitions $\forall k,\ j$ $\mathsf{E}[r_{k-j}] = R$ and $\mathsf{E}[i_{k-j}] \leq m-1$. Hence

$$\mathsf{E}[w_k^*] \geq R\sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j+1} - \frac{m-1}{m}\sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j} = R\frac{n}{m-n} - \frac{m-1}{m-n}.$$

The last expression is positive if $Rn > m$. □

Second Moment Estimate

$$\mathsf{E}\left[{w_k^*}^2\right] = \mathsf{E}\left[\left(\sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j+1} r_{k-j}\right)^2\right] - \qquad (W_1)$$

$$-\frac{2}{m}\mathsf{E}\left[\sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j+1} r_{k-j}\sum_{s=0}^{\infty}\left(\frac{n}{m}\right)^{s} i_{k-s}\right] + \qquad (W_2)$$

$$+\frac{1}{m^2}\mathsf{E}\left[\left(\sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j} i_{k-j}\right)^2\right] \qquad (W_3)$$

Second Moment Estimate

Let's $\forall k,\ j\ R^{(2)} = \mathsf{E}[r_{k-j}^2]$ and $\forall k,\ j \neq s\ R^2 = \mathsf{E}\left[r_{k-j} r_{k-s}\right]$. Then

$$W_1 = R^{(2)} \sum_{j=1}^{\infty} \left(\frac{n}{m}\right)^{2j} + 2R^2 \sum_{j=0}^{\infty} \sum_{s=j+1}^{\infty} \left(\frac{n}{m}\right)^{j+s+2} =$$

$$R^{(2)} \frac{n^2}{m^2 - n^2} + 2R^2 \frac{n^3}{(m-n)(m^2-n^2)} \tag{6}$$

Second Moment Estimate

$$W_2 = \frac{2}{m}\sum_{j=0}^{\infty}\sum_{s=0}^{\infty}\left(\frac{n}{m}\right)^{j+s+1}\mathsf{E}[r_{k-j}i_{k-s}] <$$

$$2R\sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{j+1}\left(1-\frac{n}{m}\right)^{-1} = \frac{2Rmn}{(m-n)^2} = E_l. \tag{7}$$

Furthermore $\forall k,\ j\ \mathsf{E}[i_{k-j}^2] < m^2$. Also $\forall k,\ j \neq s\ \mathsf{E}[i_{k-j}i_{k-s}] < m^2$ as well.

$$W_3 < \sum_{j=0}^{\infty}\left(\frac{n}{m}\right)^{2j} + 2\sum_{j=0}^{\infty}\sum_{s=j+1}^{\infty}\left(\frac{n}{m}\right)^{j+s} =$$

$$\frac{m^2}{m^2-n^2} + \frac{2nm^2}{(m-n)(m^2-n^2)} = E_g. \tag{8}$$

Second Moment Estimate

Theorem 3 *If*

$$\frac{n^2 R}{m+n} > m. \tag{9}$$

then the following interval estimate holds

$$W_1 - E_l < \mathsf{E}\left[w_k^{*2}\right] < W_1 + E_g. \tag{10}$$

Compare with

Altman E., Avrachenkov K., Barakat C. A Stochastic model of TCP/IP with Stationary Random Losses, Proceedings of ACM SIGCOMM'00. Stockholm, 2000. pp. 231-242.

The term W_1 coincides up to notation and the terms E_l and E_g appear due to the discrete nature of the random process considered and due to the using of the *floor* operation in the AIMD control.

Conclusion

- The stepwise model of AIMD New Reno congestion control with **arbitrary** decrease coefficient is analyzed.
- The theorem on its ergodic properties is proved.
- The estimate of stationary **expectation** of congestion window size is obtained and its applicability terms are derived.
- The interval estimates of stationary congestion window size **second moment** is obtained and its applicability terms are derived.
- Further analysis yields bounds of the standard deviation which could not be obtained through expectation estimates constructed using Goelder's inequality.