Assympotic Properties of Discrete and Pice-wise Models of Additive Increse Multiplicative Decrese Algorithm.

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Problem Statement and Topicality

- Random walks with additive increase and multiplicative deacrease are widely used in the modern networking environments
- Different variations of the algotithm are used by more than ten TCP protocol implementations.
- More sophisticated variations of the algorithm are proposed to provide distributed performance control for highly congested publish/subscribe IoT environments.
- Wide scope of the applicatons and strict demands to their performance define the topicality of modeling and analysis studies of the important properties of the random walks mentioned above.

Problem Statement and Topicality

- In many cases their key performance metrics could be described by step-wise random process with semi-markovian or renewal properties.
- Most reaseraches the step-wise process is substituted by pice-wise processes with polinomial (as usual linear) growth Meanwhile there are few works those research a connection between the step-wise and corresponding pice-wise process.
- Wide scope of the applicatons and strict demands to their performance define the topicality of modeling and analysis

Problem Statement and Topicality

- TCP CUBIC - cubical growth period. RTT independent
- High Speed TCP (HSTCP), S. Floyd 2003. Congestion Avoidance coeff. are convex functions of current window size
- Scalable TCP (STCP) T. Kelly, 2003. Decreases time of data recovery
- H-TCP, Hamilton Institute, Ireland, 2004. Intended for links with high BDP value. Uses RTT size to react on losses
- TCP Hybla 2003-04. Developed for satellite links. Scales throughput to mimic NewReno and utilize link at the same time.
- TCP-Illinois uses dynamic function for defining CA parameters
- TCP-LP (Low Priority)
- TCP-YeAH

Problem Statement and Topicality

- `Sub\Pub` IoT environment
- SIB (Semantic Broker) delivers notifications to clients on the state of their subscriptions.
- Clients implement AIMD to reduce SIB workload and battery consumption.

Two mainstream modeling approaches to TCP begavior

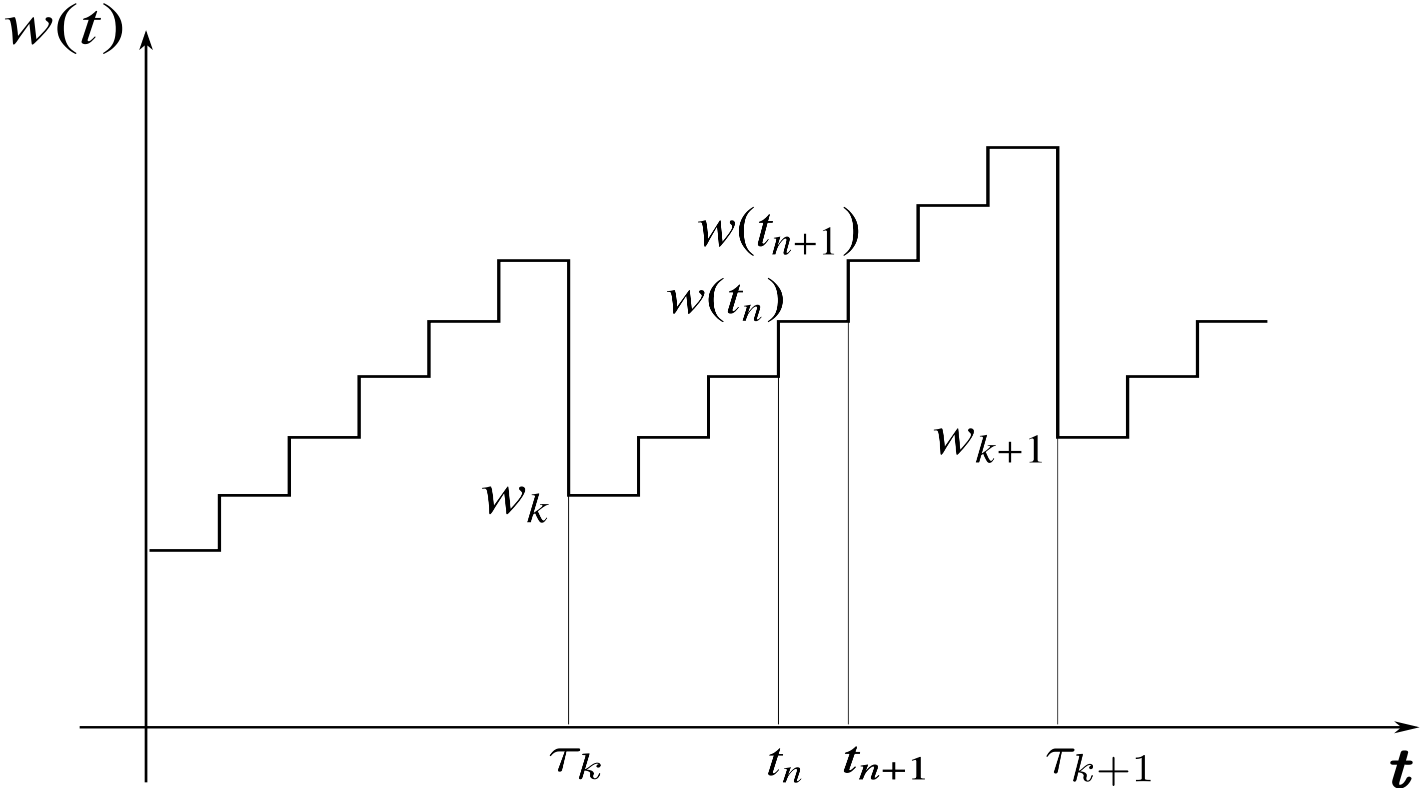

Figure 1: The step-wise random process of the congestion window size.

Two mainstream modeling approaches to TCP begavior

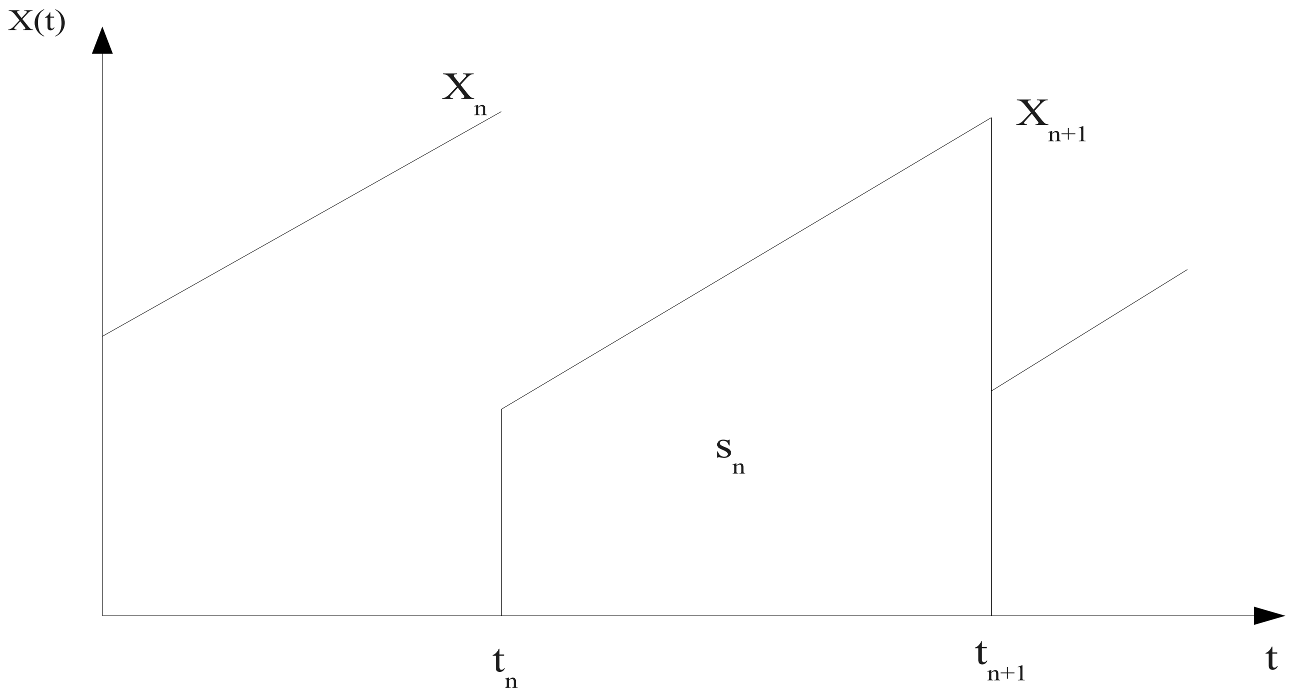

Figure 2: The piecewise linear random process of the congestion window size.

Definitions

Let's t_n — denote ends of TCP rounds
$[t_{n-1}, t_n]$ is RTT and
$\xi_n = t_n - t_{n-1}$ is RTT length.
Let's $w(t)$ — denotes cwnd.
We define stepwise process $\{w(t)\}$ such that

$$w(t_n + 0) = \begin{cases} \left\lfloor \dfrac{w(t_n)}{\alpha} \right\rfloor, & \text{if during } [t_{n-1}, t_n] \\ & \text{TCP lost data,} \\ w(t_n) + 1, & \text{if all data delivered.} \end{cases}$$

Between moments t_n the process $\{w(t)\}$ stays constant.

Definitions

Lets $\{X(t)\}_{t>0}$ takes values from $\mathbb{R}^+$ and

for the intervals $[\theta_n, \theta_{n+1})$ $n = 0, 1, \ldots$ grows linearly with the speed $b = \mathsf{E}[\xi_n]^{-1}$, . . $X(t) = X(t_0) + bt$, $\forall\ [t_0, t] \subset [\theta_n, \theta_{n+1})$.

At random moments $\{\theta_n\}_{n\geq 0}$ the process $\{X(t)\}_{t>0}$ makes a jump $X(\theta_n + 0) = X(\theta_n)/\alpha$, $\alpha > 1$.

We assume that the sequence $\{\theta_n\}_{n\geq 0}$ forms poisson flow with parameter $0 < \lambda < \infty$.

Estimates of the connection

Let us assume that the anount of data sent between two consequtive losses for i.i.d sequence of r.v.

Theorem 1 *Then for the steady state the following estimate takes place*

$$\mathsf{E}[X^2] - \frac{1}{1-\alpha^2} \leq \mathsf{E}[W^2] \leq \mathsf{E}[X^2] \qquad (1)$$

Proof

The following equation takes place

$$W_{n+1}^2 = \lfloor \alpha^2 W_n^2 \rfloor + S_n \tag{2}$$

It could be rewriten as

$$W_{n+1}^2 = \alpha^2 W_n^2 - \alpha^2 * \gamma_n + S_n, \tag{3}$$

where $0 \leq \gamma_n \leq 1$.

Proof

- The equation (2) is recurrent stochastic equation which according to Altman has stationary solution.
- Then applying expectation operation to both sides of the solution after simple transformation one gets the estimate (1).
- Let us notice that typical values of W in practice lay between 20 and 120. If $\alpha = 1/2$ wich is the standard value for TCP protocol then second term of (1) is 4.
- Piece-wiswe process provides good estimate for practical performance studies.

Conclusion

- The Development of AIMD algorithm and two main approaches to its modelling are considered.
- The theorem on the connection between parameters of such processes is prooved.
- The interval estimate obtained demonstrates that peice-wise model provides good estimate for practical performance studies.