

# ECG Feature Extraction based on Joint Application of Teager Energy Operator and Level-Crossing Sampling

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This research is financially supported by the Ministry of Education and Science of Russia within project no. 2.5124.2017 of the basic part of state research assignment for 2017–2019.

The reported study was funded by RFBR according to research project # 16-07-01289.

# Agenda

- Continuous health monitoring
- Study of arrhythmia detection algorithms within a CardiaCare project
- Arrhythmia detection algorithms heavily rely on features extracted from electrocardiogram recordings
- Teager energy operator is an easy-to-compute tool for peak emphasizing
- Level-crossing resampling allows to detect peak areas
- Joint application of Teager energy operator and level crossing sampling resulted in high QRS detection performance

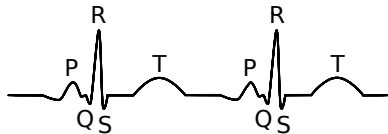


# Motivation

- 31% of all global deaths in 2012<sup>1</sup>
- Contribution of CVDs to mortality in CIS (percents)

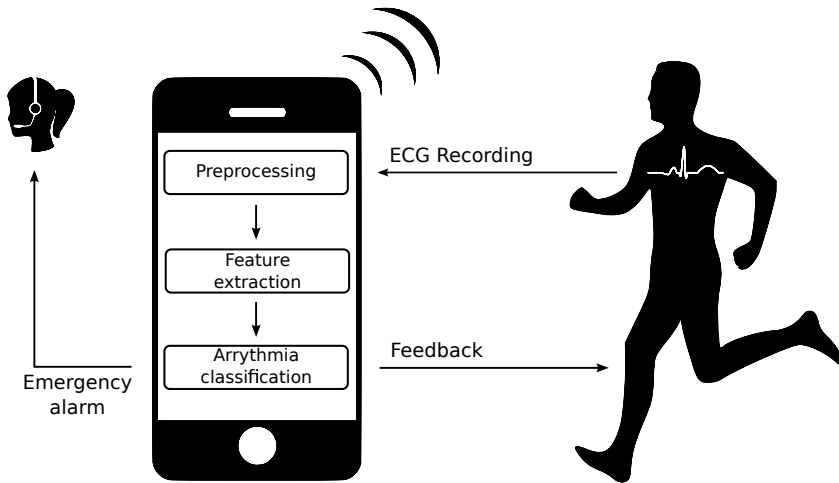
Georgia	67
Ukraine	64
Azerbaijan	60
Russia	57
Moldova	56
Belorussia	53
Kazakhstan	50
Armenia	50
Kyrgyzstan	49
Tajikistan	39

- Can be prevented by addressing behavioural risk factors (tobacco use, unhealthy diet, obesity, physical inactivity, etc.)
- Need early detection and management
- Can be done based on ECG analysis



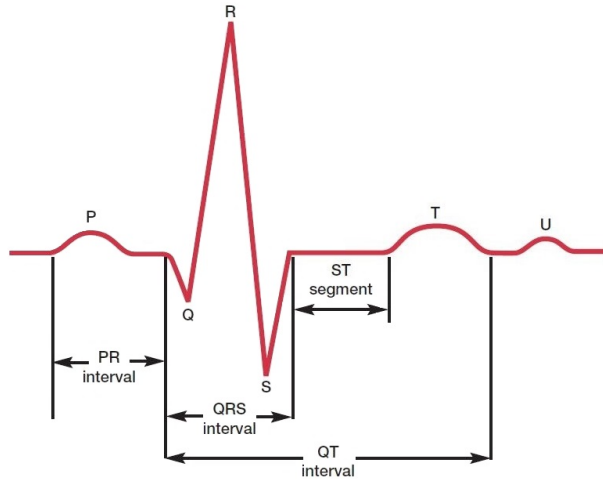
<sup>1</sup>Source: WHO

# Arrhythmia detection based on continuous monitoring



# ECG Morphology

We are interested in R peaks and QRS complexes.



# Significance of confident R peak detection

- Normal sinus rhythm



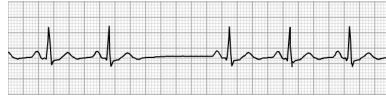
- Sinus tachycardia



- Sinus bradycardia



- Sinoatrial block



- Atrial flutter



- Wolff-Parkinson-White syndrome



Source: Medical Training and Simulation LLC  
<http://www.practicalclinicalskills.com>

# Teager-Kaiser energy operator based approach

From Hooke's law the second order differential equation can be deduced by means of Newton's second law to describe the simple harmonic motion as

$$F = \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad (1)$$

The solution to equation is given by

$$x(t) = A \cos(\omega t + \phi) \quad (2)$$

where  $x(t)$  is the position of the object at time  $t$ ,  $A$  is the amplitude,  $\omega$  is the frequency, and  $\phi$  is the initial phase. The total energy of the object is given as the sum of kinetic energy of the object and the potential energy of the spring, given by

$$E = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 \quad (3)$$

By substituting  $x(t) = A \cos(\omega t + \phi)$ , we get the following expression for the energy:

$$E = \frac{1}{2}mA^2\omega^2 \quad (4)$$

## Teager-Kaiser energy operator based approach (cont.)

Now we consider the continuous-time form of Teager energy operator defined to be

$$\Psi_c[x(t)] = (\dot{x}(t))^2 - x(t)\ddot{x}(t) \quad (5)$$

Substituting  $x(t) = A \cos(\omega t + \phi)$ , we obtain

$$\Psi_c[x(t)] = A^2\omega^2 \quad (6)$$

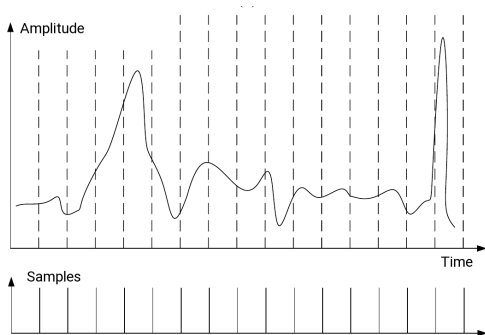
Thus, the operator defined by 5 is the amplitude and frequency product squared. But from 4 the total energy is proportional to the amplitude and frequency product squared. The discrete-time form of the Teager energy operator is defined by

$$\Psi_d[x_n] = x_n^2 - x_{n-1}x_{n+1} \quad (7)$$

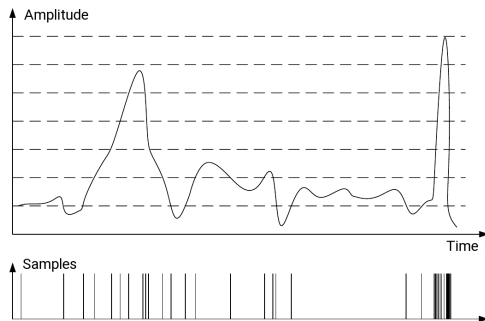


# Level-crossing sampling

## Non-uniform sampling



a) uniform sampling



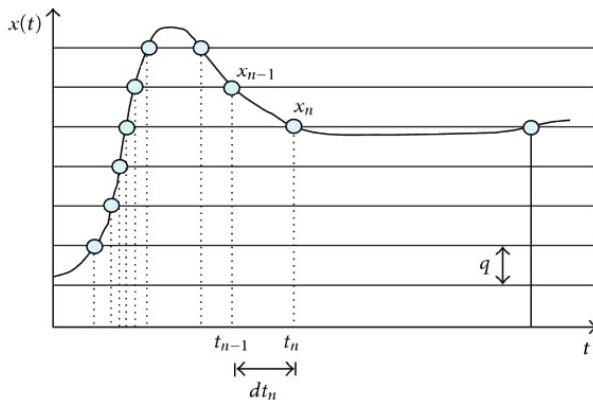
b) level-crossing sampling

Can be applied to digital signals as well!

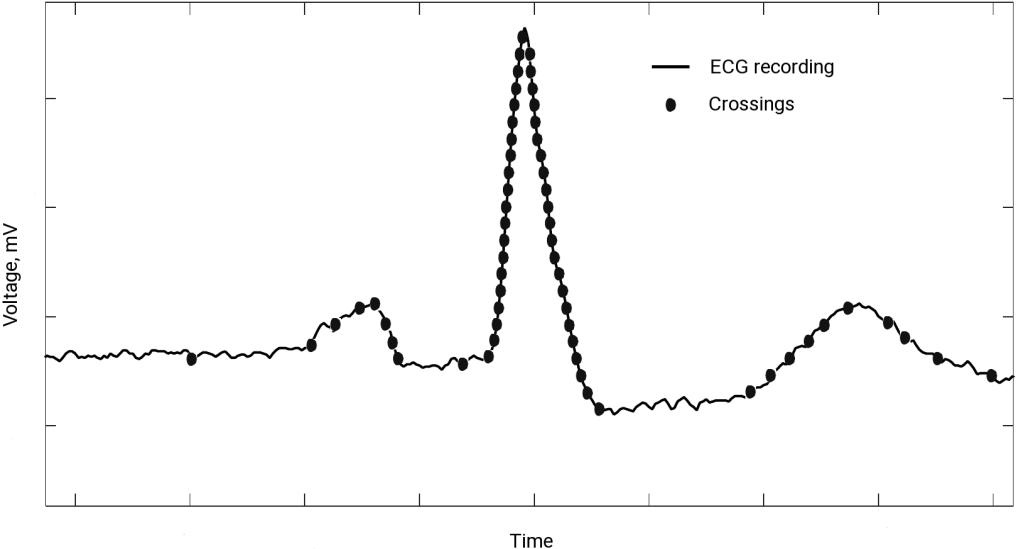
# Notation

Let  $x(t)$  be the signal. Select the levels  $\{L_1, \dots, L_m\}$  ( $\Delta L = q$ ).

Applying the method, we obtain the sequence  $\{x_1, \dots, x_n\}$  and time moments  $\{t_1, \dots, t_n\}$ . Denote the intervals  $[t_{i-1}, t_i]$  as  $dt_i$ .

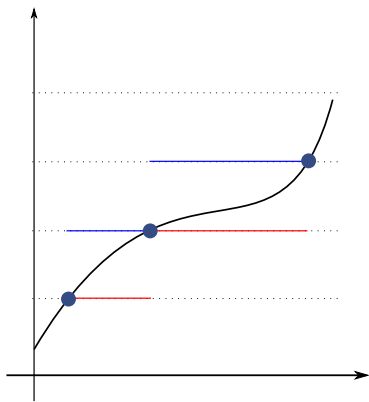


# Level-crossing for peak areas detection

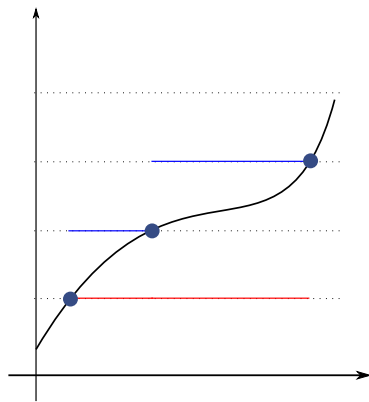


# Level-crossing with denoising

- Assume, we have  $M$  bits for a sample, then there are  $2^M - 1$  levels.
- The input signal is between  $N$  less significant bit value  $q$ .  
 $q = 2A/2^M$ .



a)  $N = 1$

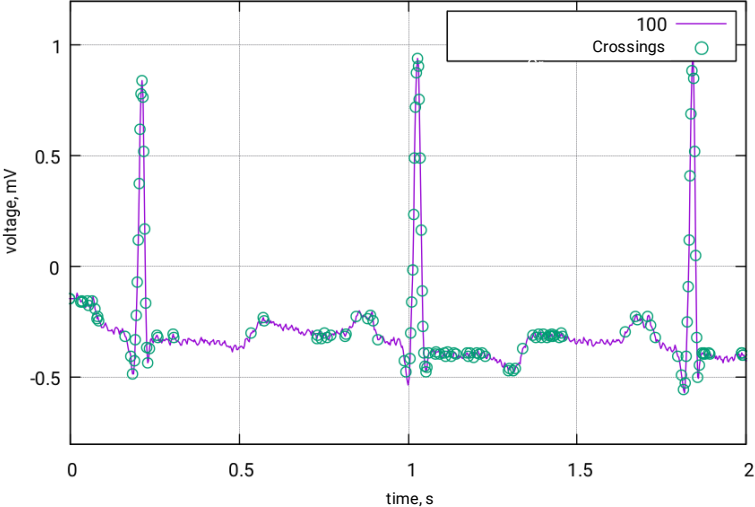


b)  $N = 2$

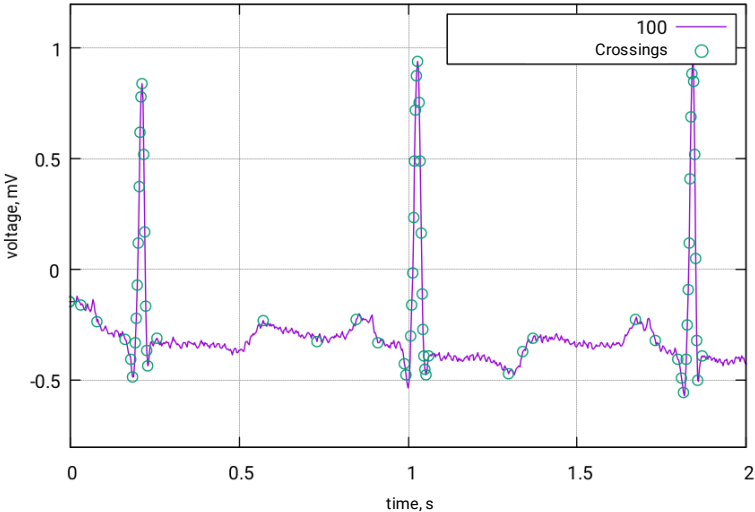
# Crossings detection

```
1 Input: ecg – digital ecg recording as the collection of pairs:
2 t – sample time
3 v – voltage
4 procedure GETCROSSINGS(ecg)
5   voltage  $\leftarrow ecg[0].v$ 
6   level  $\leftarrow \lfloor (A + \textit{voltage}) / (2 \times A) \times (2^M) \rfloor$ 
7   lower =  $q \times \textit{level} - A$ 
8   upper =  $q \times (\textit{level} + 1) - A$ 
9   for  $i \in 1 \dots ecg.size() - 1$  do
10    voltage  $\leftarrow ecg[i].v$ 
11    level  $\leftarrow \lfloor (A + \textit{voltage}) / (2 \times A) \times (2^M) \rfloor$ 
12    if voltage > upper then
13      lower  $\leftarrow q \times (\textit{level} - N + 1) - A$ 
14      upper =  $q \times (\textit{level} + 1) - A$ 
15      yield ecg[i].t
16    else if voltage < lower then
17      lower =  $LSB \times \textit{level} - A$ 
18      upper =  $LSB \times (\textit{level} + N) - A$ 
19      yield ecg[i].t
```

# Level crossings detection with no noise suppression



# Level crossings detection with noise suppression



# Interval lengths calculation

We search R-peaks among crossings  $t_k$ . Define the sliding window of  $W$  consecutive crossings

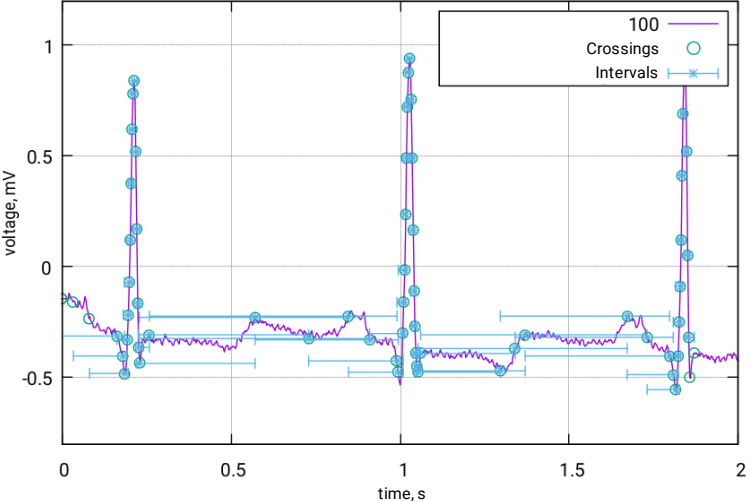
$$D(t_k) = \sum_{i=t_k - \lfloor \frac{W}{2} \rfloor}^{t_k + \lceil \frac{W}{2} \rceil - N} dt_i.$$

If  $D(t_k)$  is lesser than  $T$ , then consider  $t_k$  as a peak.

We adopt the heuristics: the QRS width-to-height ratio should be less than one tenth.



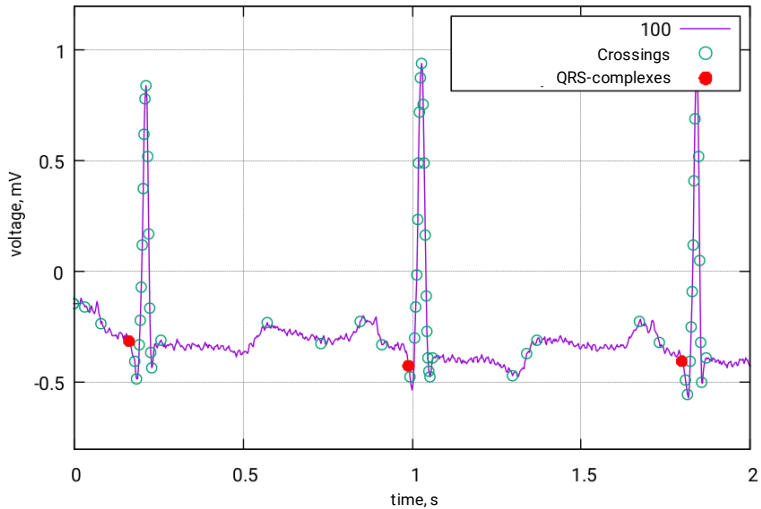
# Interval lengths example



# QRS detection algorithm

```
1  Input: seqs – sequences as the collection of triples:
2  d – sequence duration
3  t – time moment of base crossing
4  v – voltage at the base crossing
5  Thresholding values:
6   $T_{QRS}$  – maximum QRS
7   $T_V$  – the most allowed distance between crossings
8   $T_R$  – the threshold of width-to-height ratio
9  procedure GETQRS(seqs)
10 for  $i \in 1 \dots seqs.size() - 2$  do
11   if  $seqs[i].d < T_{QRS}$  and  $seqs[i - 1].d \geq T_{QRS}$  then
12      $l = i - 1$ 
13     while  $l \geq 0$  and  $seqs[l + 1].t - seqs[l + 1].t < T_V$  do
14        $l \leftarrow l - 1$ 
15      $r = i + 1$ 
16     while  $r < seqs.size() - 1$  and  $seqs[r].t - seqs[r - 1].t < T_V$  do
17        $r \leftarrow r + 1$ 
18      $mx = \max\{seqs[i].v \mid \forall i \in [l \dots r]\}$ 
19      $mn = \min\{seqs[i].v \mid \forall i \in [l \dots r]\}$ 
20     if  $(mx - mn) / (seqs[r].t - seqs[l].t) > T_R$  then
21       yield ( $l, r$ )
```

# QRS detection example



# QRS detection performance

With the heuristics:

$$\begin{aligned} \textit{Precision} &= 94,6 \% \\ \textit{Recall} &= 90,6 \% \\ \textit{F - measure} &= 92,3 \% \end{aligned}$$

With the Teager energy operator support:

$$\begin{aligned} \textit{Precision} &= 97,4 \% \\ \textit{Recall} &= 94,8 \% \\ \textit{F - measure} &= 96,1 \% \end{aligned}$$

The proposed method has the following advantages:

- extremely low computational complexity;
- high performance have been proven on MIT-BIH database.

Disadvantages:

- considerable performance decrease on very noisy signals.