Convergence of Discrete Models of TCP Congestion Avoidance.

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TCP Congestion Control

- The paradigm of Distributed Control in Packet Switching Network
- Transmission Control Program, 1974.
- Congestion collapse
- Variance not important yet

TCP Congestion Control Development

- Jitter sensitive applications
- TCP vs UDP
- High BDP links utilization vs Congestion Control
- Best effort vs QoS guarantees

Variety of Experimental Versions

- TCP CUBIC - cubical growth period. RTT independent
- High Speed TCP (HSTCP), S. Floyd 2003. Congestion Avoidance co-eff. of linear growth and multiplicative decrease are convex functions of current window size
- Scalable TCP (STCP) T. Kelly, 2003. Decreases time of data recovery
- TCP Hybla 2003-04. Developed for satellite links. Scales throughput to mimic NewReno and utilize link at the same time.
- TCP-YeAH

Two mainstream modeling approaches to TCP begavior

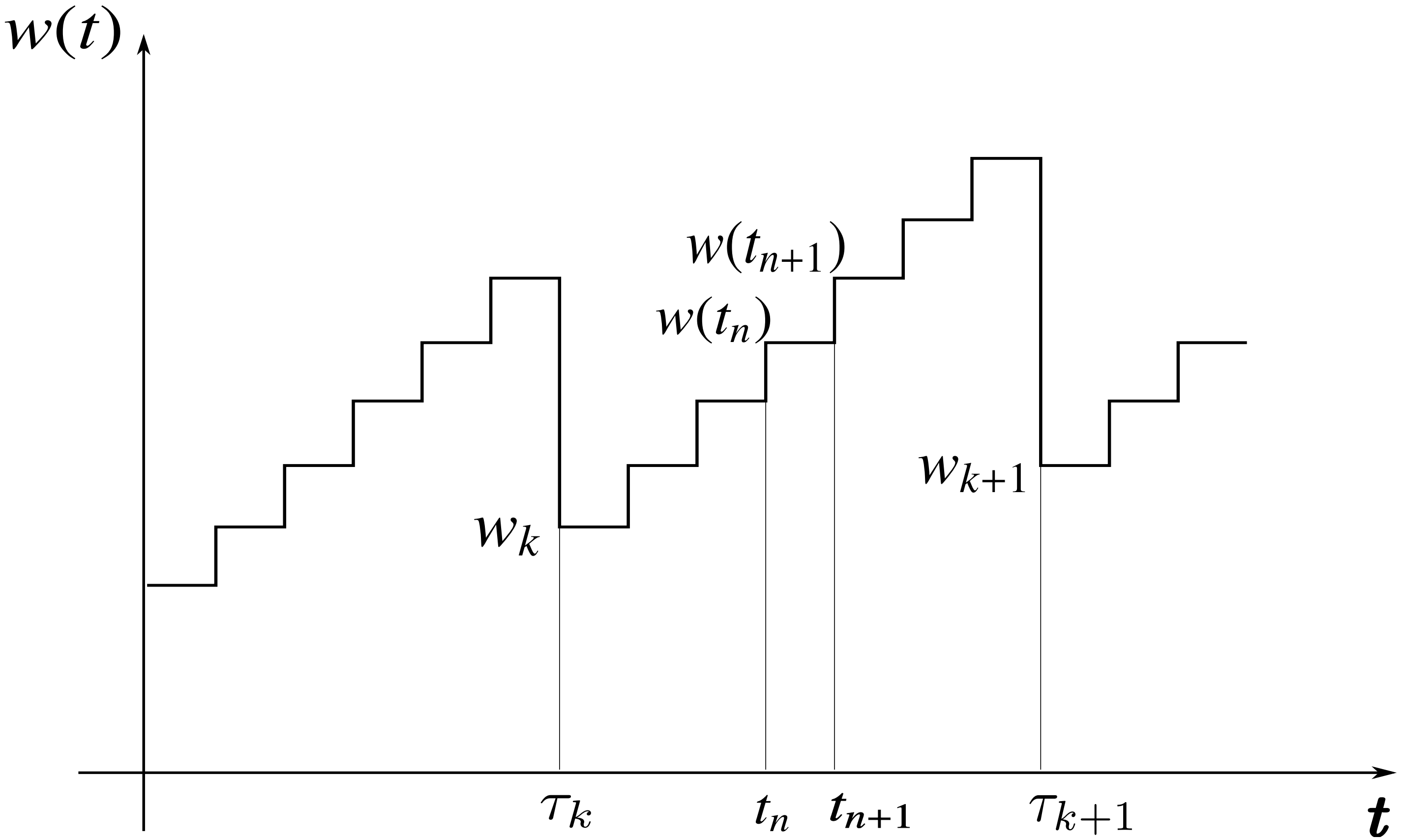

Figure 1: The step-wise random process of the congestion window size.

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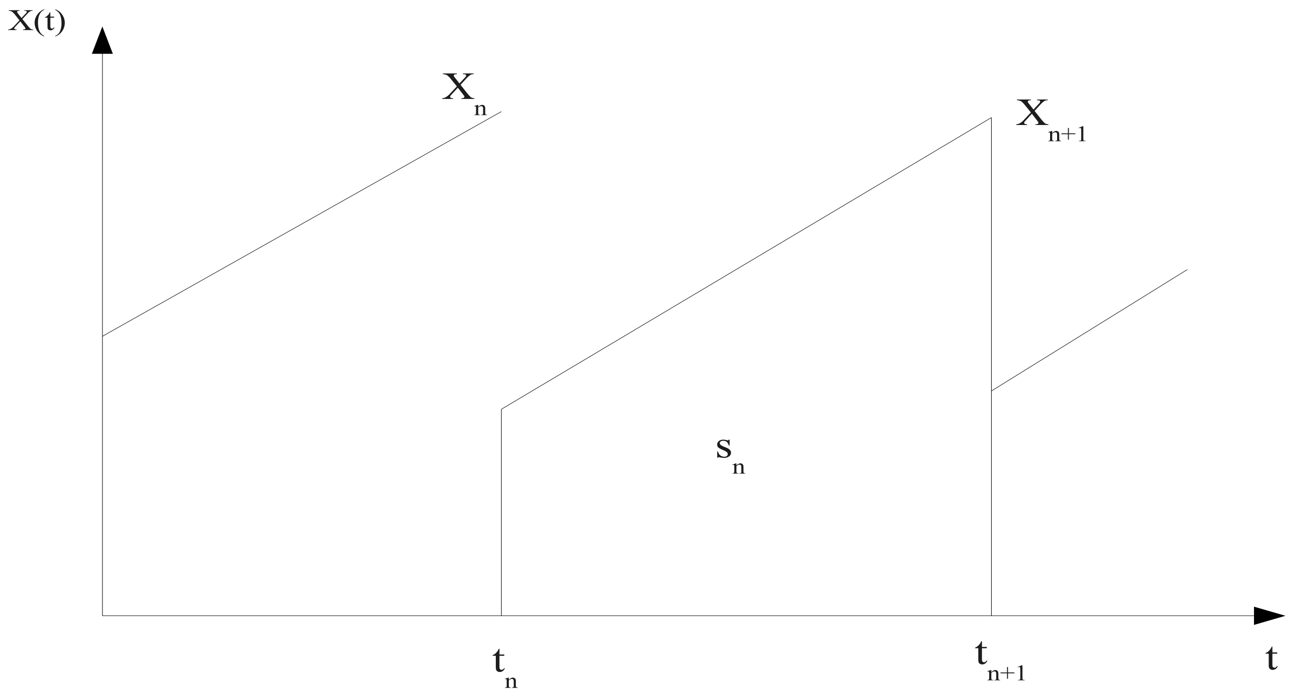

Figure 2: The piecewise linear random process of the congestion window size.

Definitions

Let's t_n — denote ends of TCP rounds
$[t_{n-1}, t_n]$ is RTT and
$\xi_n = t_n - t_{n-1}$ is RTT length.
Let's $w(t)$ — denotes cwnd.
We define stepwise process $\{w(t)\}$ such that

$$w(t_n + 0) = \begin{cases} \left\lfloor \dfrac{w(t_n)}{\alpha} \right\rfloor, & \text{if during } [t_{n-1}, t_n] \\ & \text{TCP lost data,} \\ w(t_n) + 1, & \text{if all data delivered.} \end{cases}$$

Between moments t_n the process $\{w(t)\}$ stays constant.

Definitions

Lets $\{X(t)\}_{t>0}$ takes values from $\mathbb{R}^+$ and

for the intervals $[\theta_n, \theta_{n+1})$ $n = 0, 1, \ldots$ grows linearly with the speed $b = \mathsf{E}[\xi_n]^{-1}$, . . $X(t) = X(t_0) + bt$, $\forall$ $[t_0, t] \subset [\theta_n, \theta_{n+1})$.

At random moments $\{\theta_n\}_{n\geq 0}$ the process $\{X(t)\}_{t>0}$ makes a jump $X(\theta_n + 0) = X(\theta_n)/\alpha$, $\alpha > 1$.

We assume that the sequence $\{\theta_n\}_{n\geq 0}$ forms poisson flow with parameter $0 < \lambda < \infty$.

Convergence Theorem

We propose following transformation of coordinates for the process $\{w(t)\}$

$$t = ns \qquad w = \lfloor nX \rfloor. \tag{1}$$

Lets consider following sequence of stepwise processes $w_n(s) = w(ns)$: $\lambda_n = \lambda/n$. We denote $w_n(0) = \lfloor nx_0 \rfloor \;\; X(0) = x_0$.

Theorem 1 $\forall$ s *takes place*

$$\lim_{n\to\infty} \frac{w(ns)}{n} = X(s) \tag{2}$$

by distribution. And $b = \left[\int\limits_0^\infty x dR(x) \right]^{-1}$.

Proof

Let us consider a growth period of $X(s)$. For the interval $[s_1, s_2], \subset [\theta_n, \theta_{n+1}]$ it takes place $X(s) = X(s_1) + b(s - s_1)$.

Let's denote $u_n(s, s_1)$ the number of the moments t_m, happened in the interval $[ns_1, ns]$. The sequence $\{t_k\}_{m=1}^{\infty}$ makes renewal process and according to Smith theorem

$$\lim_{n\to\infty} \frac{\mathbf{E}[u_n(s, s_1)]}{n(s - s_1)} = b \qquad (3)$$

Then according to Chebyshev inequality

$$\lim_{n\to\infty} \frac{u_n(s, s_1)}{n} = b(s - s_1) \qquad (4)$$

by probability.

Proof

Now denote $w_{n,k} = w_n(\tau_k - 0)$ and $j_{n,k} = u_n(\sigma_{n,k}, \sigma_{n,k-1})$, where $\tau_k = n\sigma_{n,k}$.

$$w_{n,k} = \left\lfloor \frac{w_{n,k-1}}{\alpha} \right\rfloor + j_{n,k} = \frac{w_{n,k-1}}{\alpha} - \gamma_{n,k} + j_{n,k}, \tag{5}$$

where $0 \leq \gamma_{n,k} < 1$.

The interval $\tau_k - \tau_{k-1} = n(\sigma_{n,k} - \sigma_{n,k-1}) = \pi_k + \delta_k$, where $\eta_k = \pi_k/n$, has distribution $F_{\eta_k}(x) = 1 - e^{-\lambda s}$,
and r.v. δ_k/n converges by probability to zero.

Henceforth

$$\lim_{n\to\infty}(\sigma_{n,k} - \sigma_{n,k-1}) = \eta_k. \tag{6}$$

by distribution.

Proof

Let $n\theta'_n$ — the moment of the last jump of $w_n(s)$ before moment ns, ν_n — its number and $w_n(0) = j_0^n$. then

$$w_n(s) = \frac{1}{\alpha}\left[\sum_{i=0}^{\nu_n} \frac{j_{n,\nu_n - i}}{\alpha^i} - \sum_{i=0}^{\nu_n} \frac{\gamma_{n,\nu_n - i}}{\alpha^i}\right] + u_n(s - \theta'_n). \qquad (7)$$

From (6) one infers that $\nu_n \to \nu$, by probability and ν satisfies Poisson distribution.

Also $\theta'_n \to \theta'$ by distribution too, and θ' is the moment of the last jump of the process $X(s)$ before time moment s. Hence

$$\lim_{n\to\infty} \frac{w(ns)}{n} = \frac{b}{\alpha}\sum_{i=0}^{\nu} \frac{\eta_{\nu-i}}{\alpha^i} + b(s - \theta') = X(s) \qquad (8)$$

by distribution, where $\eta_0 = x_0$.

Conclusion

- The Development of Congestion Control schemes and two main approaches to its modelling are considered.
- The sequence of the stepwise AIMD models is built.
- For the sequence the convergence theorem is proved.
- Further development: investigate the speed of the convergence.