Verification of communication node effective bandwidth estimator

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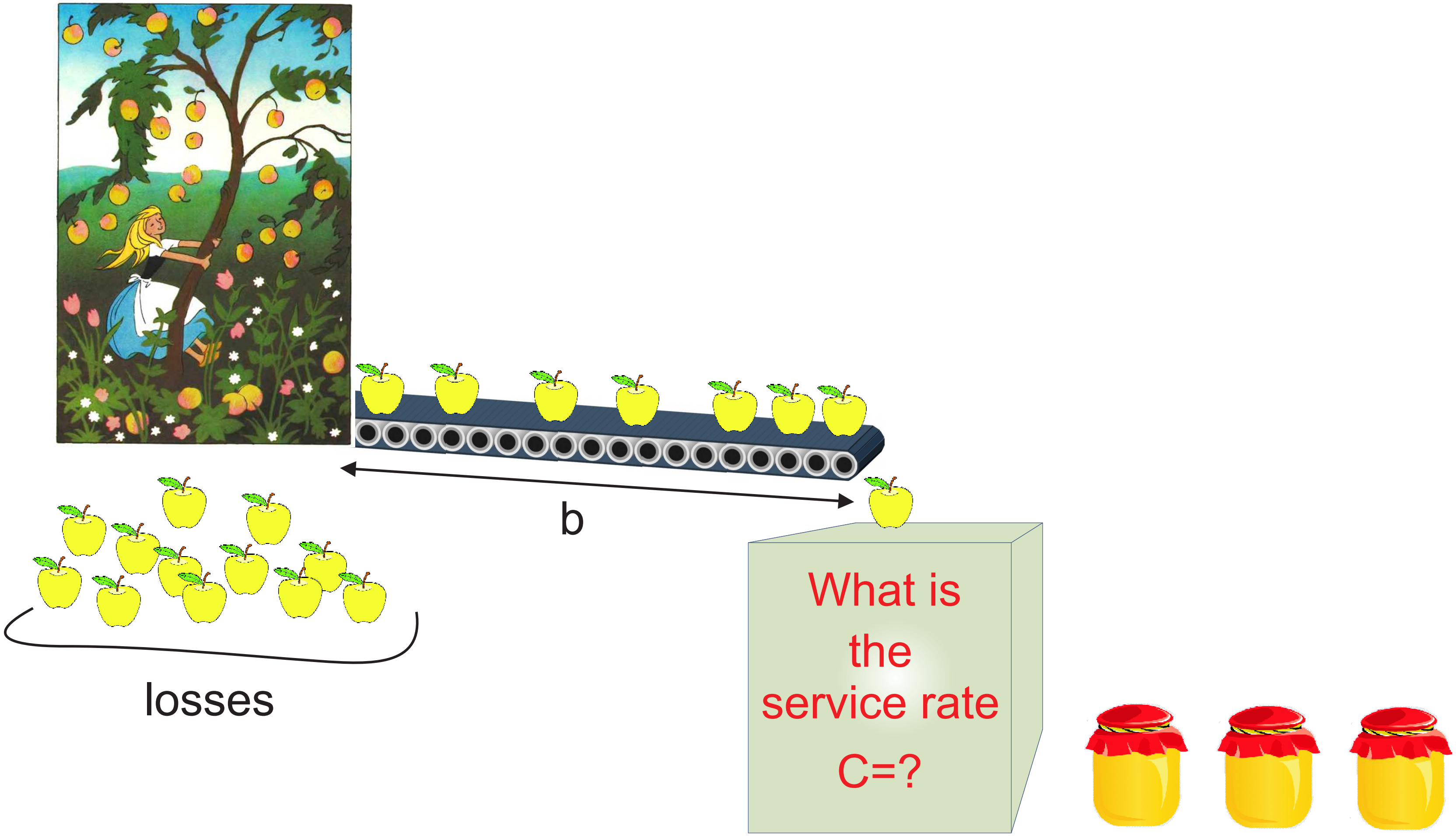
Nature of the problem



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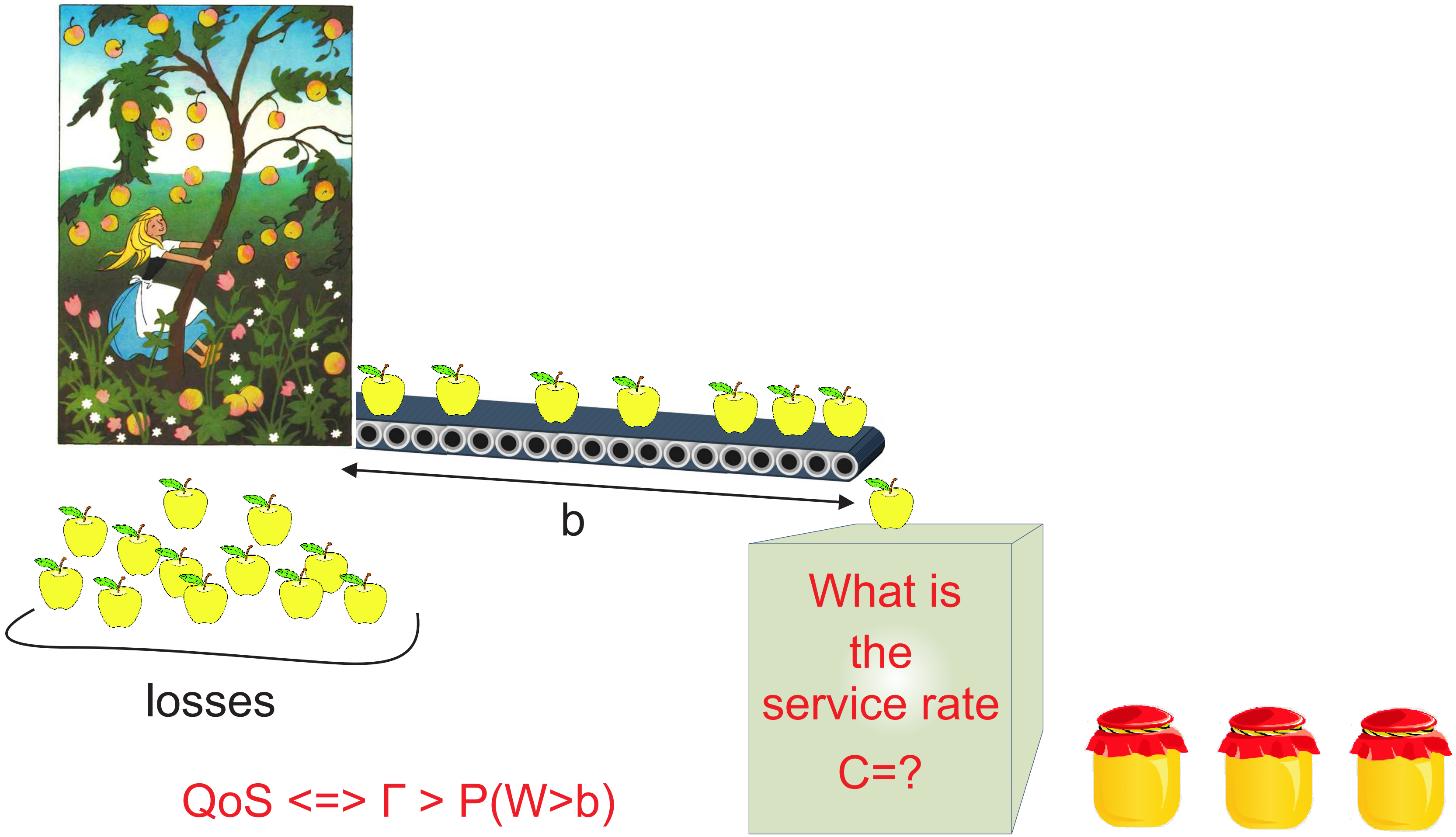
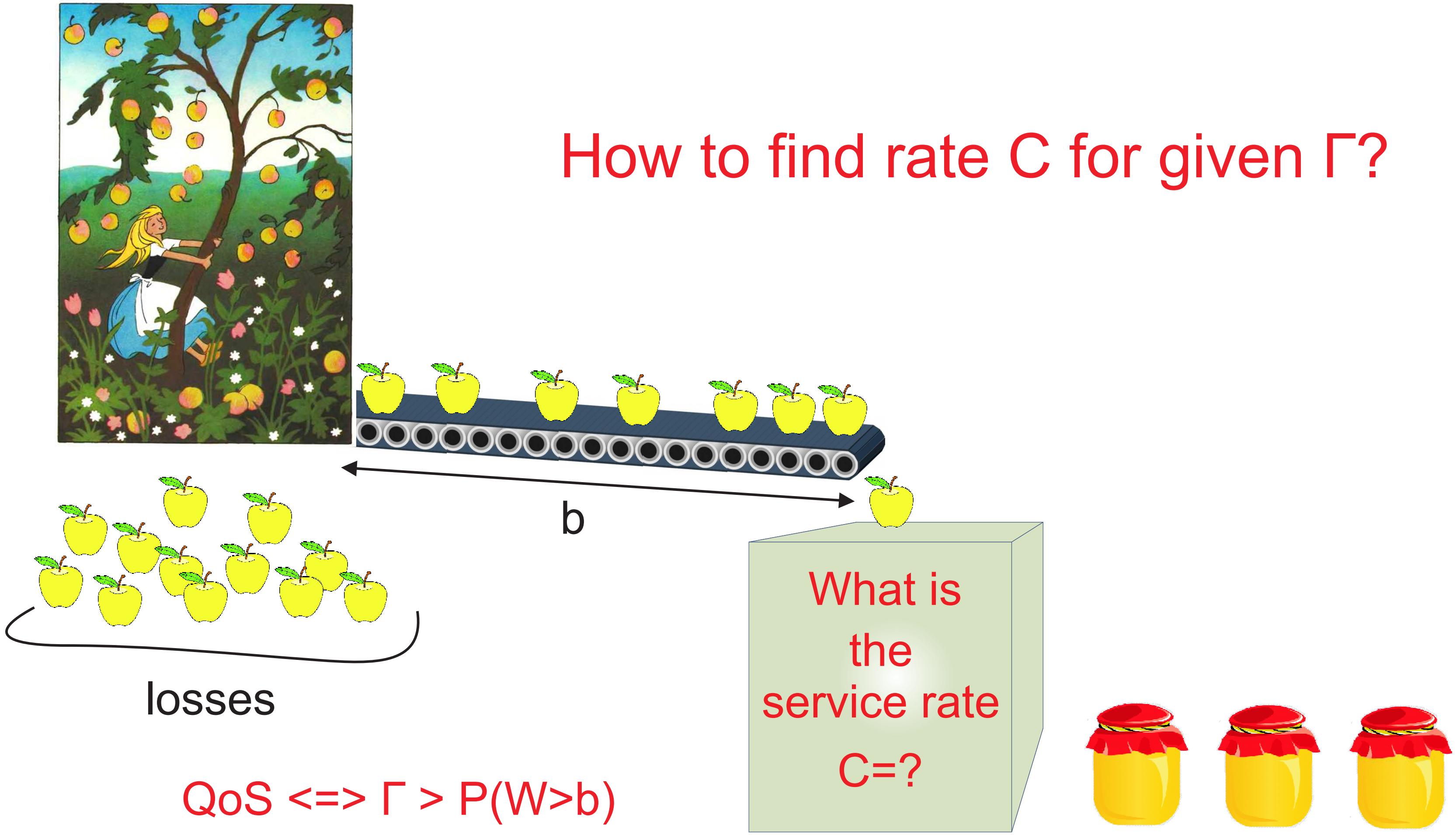


Nature of the problem

How to find rate C for given Γ?



Effective bandwidth problem

Consider a buffered queue with a positive recurrent regenerative input and constant service rate C. The **effective bandwidth (EB) problem** is to find the *minimal rate* C that allows to guarantee given QoS level Γ for overflow/loss probability

$$\mathsf{P}_b = \mathsf{P}(W > b) \leq \Gamma, \tag{1}$$

where W is stationary workload process, b the buffer size.

An exponential approximation for W follows from **Large Deviation Principle**

$$\mathsf{P}_b \asymp e^{-\theta^* b}, b \to \infty, \tag{2}$$

where $\asymp$ means *logarithmic asymptotics.*

Then (1), (2) define unknown guarantee parameter

$$\theta^* = -\ln \Gamma / b > 0. \tag{3}$$

EB definition

Frank Kelly (1991), Ward Whitt (1993), G. de Veciana и J. Walrand (1995)

Determine the limiting scaled *cumulant generating function* of the input process

$$\Lambda(\theta) = \lim_{n\to\infty} \frac{1}{n} \log \mathsf{E} e^{\theta \sum_{i=1}^{n} v_i}, \tag{4}$$

where v_i denotes the amount of work that arrives per time unit $(i-1, i]$. Assuming the existence of the finite limit (4) in a neighborhood of $\theta \in (0, \theta_0)$, the EB is defined by

$$C := \frac{\Lambda(\theta^*)}{\theta^*}. \tag{5}$$

The main problem is: an analytical form (4) is difficult and sometimes impossible to find. EB estimation problem reduced to $\Lambda(\theta^*)$ estimation

Estimation of $\Lambda(\theta^*)$

Case 1: r. v. $\{v_i\}$ are i. i. d. Let $\mathsf{E}e^{\theta^* v} < \infty$, then the target (unbiased) estimator of $\Lambda(\theta^*)$ is sample mean

$$\ln \frac{1}{k} \sum_{i=1}^{k} e^{\theta^* v_i} \to \Lambda(\theta^*) = \ln \mathsf{E}e^{\theta^* v}, \; k \to \infty \text{ w. p. 1.} \tag{6}$$

Case 2: if r. v. $\{v_i\}$ are dependent there are two simulation methods for $\Lambda(\theta^*)$ estimation:

1. traditional batch means method (BM);
2. regenarative approach (REG).

The main properties

It is important to study the properties of the estimators:

- *the strong consistency* (it is obviously for BM and still the open problem for REG);
- *the bias* (this property influences whether the estimator ensures the given QoS level Γ).

Batch means method [BM]

Idea: Data from the single simulation run divided into blocks of **fixed length** B

$$\hat{X}_j = \sum_{i=(j-1)B+1}^{jB} v_i, \quad j \geq 1.$$

Main assumption: if B is large enough then r. v. $\hat{X}_j$ can be approximately regarded as i. i. d.

The BM estimator of $\Lambda_V(\theta^*, B) = \dfrac{\ln \mathsf{E} e^{\theta^* \hat{X}}}{B}$ is

$$\hat{\Lambda}_k(\theta,\ B) := \frac{1}{B} \ln \frac{1}{k} \sum_{i=1}^{k} e^{\theta \hat{X}_i} \to \Lambda(\theta^*, B),\ \ k \to \infty, \tag{7}$$

where k is the block number, $n = kB$ is the total number of observations.

BM estimator problems

1. Partition into blocks excluding properties of the input process looks quite "rough".

2. The problem is how to choose block size B to obtaine effective estimation.

3. The estimator is biased, moreover,

$$\mathsf{E}\left[\hat{C}_k(\theta^*, B)\right] < \frac{1}{\theta^* B} \ln \mathsf{E}\left[e^{\theta^* \hat{X}}\right] = C(\theta^*, B), \tag{8}$$

   so, there is a risk to choose too small rate C that doesn't provide the required guarantees for P_b.

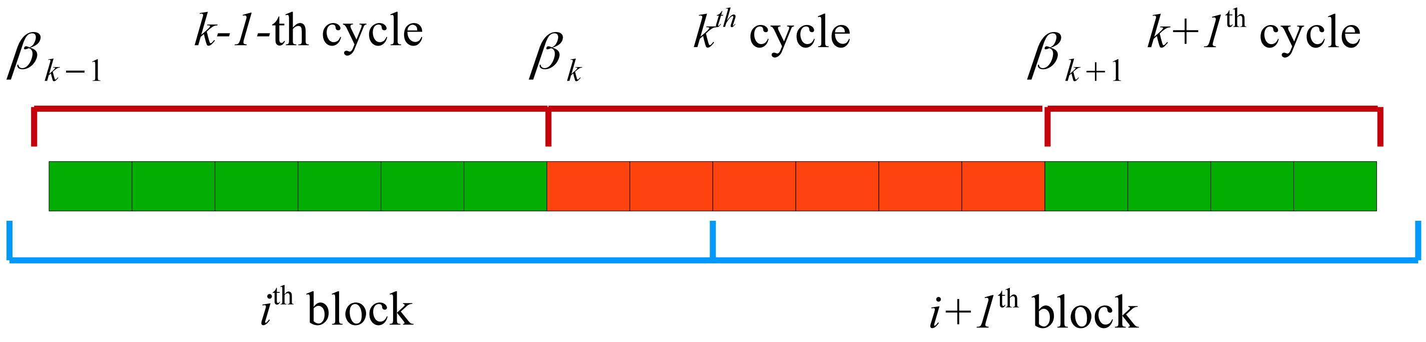
4. Due to "roughly division" dependent data can get into different blocks. This fact in turn can dramatically affect the estimator variance.

Regenerative approach

Idea: block = regenerative cycle.

Assume that the process $\{v_n, n \geq 1\}$ is regenerative, let β_k be the kth reg. time, then $\alpha_k = \beta_{k+1} - \beta_k$ is kth reg. period (cycle length). The structure of dependencies between $\{v_i\}$ can be considered in **refined EB estimator** due to division into cycles. So, the regenerative blocks are really i. i. d.

$$\hat{X}_k := \sum_{i=\beta_k}^{\beta_{k+1}-1} v_i, \; k \geq 0, \; \beta_0 = 0. \tag{9}$$



Regenerative EB estimator

Assume that $\mathsf{E}\alpha < \infty$, $\ln \mathsf{E} e^{\theta^* \hat{X}} < \infty$, $\theta^* \in (0, \theta_0)$, $\mathsf{E}(\alpha - \mathsf{E}\alpha)^2 := \sigma^2 \in (0, \infty)$, then the REG estimator of $\Lambda(\theta^*)$ defined by k regenerative cycles and w. p. 1 holds

$$\hat{\Lambda}_k(\theta^*) := \frac{k}{\beta_k} \ln \frac{1}{k} \sum_{i=1}^{k} e^{\theta^* \hat{X}_i} \to \frac{1}{\mathsf{E}\alpha} \ln \mathsf{E} e^{\theta^* \hat{X}} =: \Lambda_{REG}(\theta^*), \quad k \to \infty. \tag{10}$$

It is **necessary to prove** that the following convergence holds as $n \to \infty$

$$\frac{1}{n} \ln \mathsf{E} e^{\theta^* \sum_{i=1}^{n} v_i} \to \frac{1}{\mathsf{E}\alpha} \ln \mathsf{E} e^{\theta^* \hat{X}} = \Lambda_{REG}(\theta^*). \tag{11}$$

If so then the EB estimator can be obtained from (5) as

$$\hat{C}_k(\theta^*) = \frac{\hat{\Lambda}_k(\theta^*)}{\theta^*}. \tag{12}$$

The upper bound problem

The **lower bound** has been established in [A. Borodina, I. Dudenko, E. Morozov, 2009]

$$\lim_{n\to\infty} \inf \frac{1}{n} \mathsf{E} e^{\theta^* \sum_{i=1}^n v_i} \geq \Lambda_{REG}(\theta^*) := \frac{1}{\mathsf{E}\alpha} \ln \mathsf{E} e^{\theta^* \hat{X}}. \tag{13}$$

The **upper bound** evaluation is still the open problem

$$\lim_{n\to\infty} \sup \frac{1}{n} \mathsf{E} e^{\theta^* \sum_{i=1}^n v_i} \leq \Lambda_{REG}(\theta^*), \tag{14}$$

but we can offer the regenerative estimator as an approximation for $\Lambda(\theta^*)$

Due to simulation we were able to show that the regenarative method gives the **the pessimistic** EB estimator!

The main question is

How can we check the quality of estimation?

Means of verification

1. to calculate directly the function $\Lambda(\theta^*) = \lim_{n\to\infty} \frac{1}{n} \log \mathsf{E} e^{\theta^* \sum_{i=1}^{n} v_i}$;

2. to estimate the probability $\mathsf{P}_b = \mathsf{P}(W > b) \leq \Gamma$ for a given value of $\hat{C}$ for the stationary workload process W.

But the value of Γ is small (due to QoS requirements), so the standard Monte-Carlo method most often gives $\hat{P}_b = 0$!

Possible solutions are:

1. waiting for a long time by Monte-Carlo;

2. speed-up simulation by Splitting method (rare event simulation).

Idea of the Splitting method

We will consider Lindley's reqursion for the workload proces

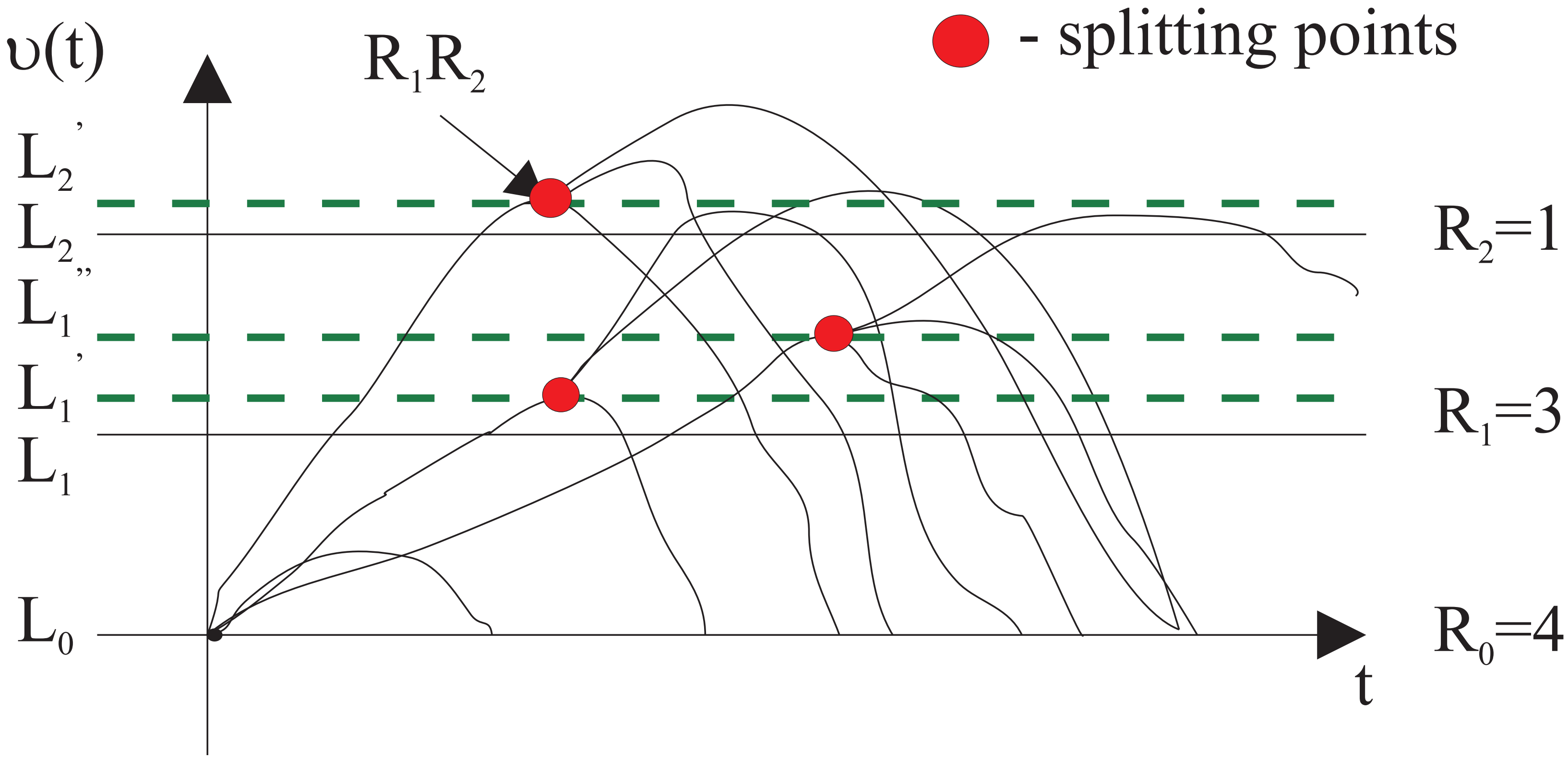
$$W_{n+1} = [W_n + v_{n+1} - C]^+,\ n \geq 0; \quad W_0 = 0, \tag{15}$$

constructed by the arrival times $\{t_n\}$, where W_n is the waiting time of the customer n in the queue.

Define the set of thresholds $L_1 \dots L_M$, $L_0 = 0$, $L_{M+1} = b$, where we will split the trajectory of the process.

Splitting condition: if the trajectory of the process hits the threshold L_{i+k}, $i+k \leq M+1$ (it happens at arrival instants) then it split into $\prod_{j=1}^{k} R_{i+j}$ subpaths.

Illustration of splitting

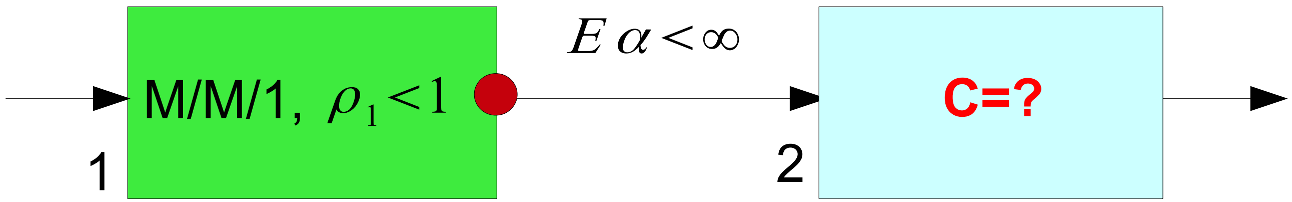


[1] A. Borodina. PhD thesis "Regenerative modification of splitting method for overload probability estimation in queuing systems" (in Russian), 2008.

Simulation: EB estimation in 2-nd node

Consider 2-node tandem network. 1-st node input process is renewal with intensity λ and i. i. d. service times $\{S, S_n\}$ with $\mathsf{E}S = 1/\mu$ and $\rho := \lambda/\mu < 1$. So, the 2-nd node is fed by a positive recurrent regenerative input, $\mathsf{E}\alpha < \infty$.

Regeneration occurs when *the 1-st node have been left by the customer which have seen the 1-st node empty.*



Verifacation via overfull probability simulation

Regenerative EB estimator for 2-node tandem Let v_i is strongly dependable on the cycle $v_j = \frac{\sum_{k=1}^{j} \eta_k}{j}$, $1 \le j \le \alpha$, where η_k distributed by Weibull ($\gamma = 3$, c=4). $\Delta := \Gamma - \hat{\Gamma}$.

#	Γ	θ^*	$\hat{C}(k)$	$\hat{\Gamma}$	Δ/Γ
1	10^{-3}	0,230259	0,264602	$8,15 \cdot 10^{-4}$	0,15
2	10^{-4}	0,307011	0,290134	$2,05 \cdot 10^{-5}$	0,75
3	10^{-5}	0,383764	0,348517	$1,84 \cdot 10^{-6}$	0,816
4	10^{-6}	0,460517	0,527721	$2,97 \cdot 10^{-8}$	0,97
5	10^{-7}	0,53727	0,661887	$0,45 \cdot 10^{-8}$	0,955
6	10^{-8}	0,614023	0,986111	$8,67 \cdot 10^{-10}$	0,913

Discrete time. Workload restrictions

Regenerative EB estimator for 2-node tandem with restrictions

#	Γ	θ^*	d	$\hat{\alpha}$	$\hat{C}(k)$	$Var\hat{C}(k)$	$\hat{\Gamma}$	Δ/Γ
1	10^{-4}	0,153506	50	89,1	0,560441	$5,23 \cdot 10^{-6}$	$0,3433 \cdot 10^{-5}$	0,6567
2	10^{-5}	0,191882	50	89,2	0,560947	$7,73 \cdot 10^{-6}$	$0,4153 \cdot 10^{-5}$	0,5847
3	10^{-6}	0,230259	70	124,9	0,561252	$2,64 \cdot 10^{-6}$	$0,8698 \cdot 10^{-6}$	0,1302
4	10^{-7}	0,268635	70	124,8	0,562472	$4,23 \cdot 10^{-6}$	$0,8871 \cdot 10^{-7}$	0,1129
5	10^{-8}	0,307011	70	124,5	0,563537	$6,98 \cdot 10^{-6}$	$0,2116 \cdot 10^{-8}$	0,7884

Thank you for attention!