Evaluation of variance for TCP throughput

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TCP Congestion Control

- In distributed packet switching network a sender is not aware about the workload experienced by key elements of the networking environment (links and routers)
- Distrubuted flow control is one of the basic networking problems
- Congestion collapse

TCP Congestion Control

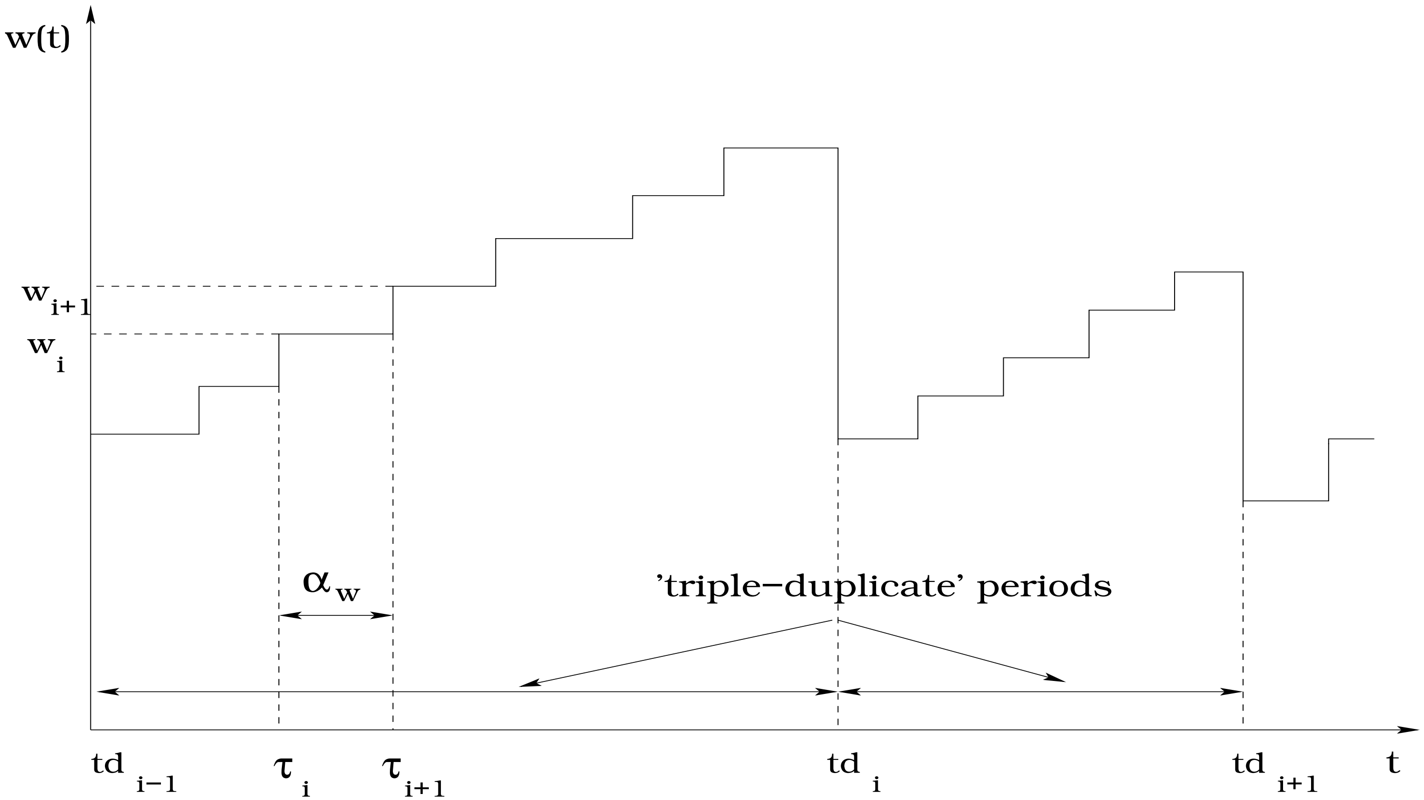
- End-to-end transport layer decides which data and when are to be injected in the network and provides delivery control as well
- TCP vs UDP
- Diversification of the networking applications and the bearers of the signal provide more sophisticated requirements to the flow control algorithms
- Reno and NewReno performance defines model behavior for TCP-friendliness (BIC, CUBIC, HTCP, etc)

Distributed Flow Control

- Data delivery control is done by **sliding window** and explicit **acknowledgme**
- Sliding window is amount of data which sender is allowed to inject in the network without acknowledgment
- Flow control means control on sliding window size. TCP uses set of algorithms to control its window size W

Additive Increase Multiplicative Decrease Algorithm (AIMD)

$$\begin{array}{lcl} W & \xrightarrow{\text{delivery}} & W+1 \\ \downarrow \text{loss} & & \\ W/2 & & \end{array}$$



TCP NewReno 'Saw'

'Root Square' Lows

Let us denote

- p TCP segment loss probability
- RTT Round Trip Time
- RTO Round Trip Timeout
- MSS Minimal Segment Size

S. Floyd for $p \leq 0.025$

$$T = \frac{MSS}{RTT}\sqrt{\frac{3}{2p}} \tag{1}$$

D. Towsley group for $p > 0.025$

$$T = \frac{MSS}{RTT\sqrt{\frac{2p}{3}} + RTOmin(1, 3\sqrt{\frac{3p}{8}})p(1 + 32p^2)} \tag{2}$$

Variance evaluation

- Throughput variance is nowadays important for many networking applications
- Variance is $Var[X] = E[X^2] - (E[X]^2)$
- Root square lows are derived thorough Goelders's inequality i.e.

$$E[X] \leq \sqrt{E[X^2]}$$

Variance evaluation

- $X_{n+1}^2 = \alpha X_n^2 + 2bS_n$, where $0 < \alpha < 1$, b - is growth ratio and S_n is amount of data sent without loss

- Expanding, one gets

$$X_{k+n}^2 = 2b \sum_{i=0}^{n} \alpha^{2i} S_{k+n-i}$$

  or

- $X_{k+n} = \sqrt{2b \sum_{i=0}^{n} \alpha^{2i} S_{k+n-i}}$

Variance evaluation

$$E[X_{k+n}] = E\left[\sqrt{2b\sum_{i=0}^{n}\alpha^{2i}S_{k+n-i}}\right] \leq \sqrt{2b}\sum_{i=0}^{n}E\left[\sqrt{\alpha^{2i}S_{k+n-i}}\right]$$

Transforming to stationary state one gets

$$E[X] = \lim_{n\to\infty}E[X_n] \leq \lim\sqrt{2b}\sum_{i=0}^{n}E\left[\sqrt{\alpha^{2i}S_i}\right] =$$

$$\frac{\sqrt{2b}}{1-\alpha}E[\sqrt{S_n}]$$

For $b = 1$ and $\alpha = \frac{1}{2}$ one gets

$$E[X] \leq 2\sqrt{2}E[\sqrt{S_n}]$$

Variance evaluation

Now we have two estimations of sliding window size expectation

- $E[X] \leq A = \sqrt{\frac{8}{3}E[S_n^2]}$
- $E[X] \leq B = 2\sqrt{2}E[\sqrt{S_n}]$

If $B < A$ then it could be used for estimation variance

$$Var[X] \leq A^2 - B^2.$$

This holds if

$$E[\sqrt{S_n}] < \sqrt{\frac{E[S_n]}{3}}.$$

Variance evaluation. Examples

Lets p.d.f. of S_n is $F(x) = 1 - e^{\lambda x}$ then

$$E[S_n] = \frac{1}{\lambda}$$

and

$$E[\sqrt{S_n}] = \frac{\sqrt{\pi}}{2}\sqrt{\frac{1}{\lambda}}.$$

Condition does not hold.

Variance evaluation

Lets p.d.f of S_n is Pierson root square distribution then its moments can be calculated through Γ-function and

$$E[S_n] = 2^{\frac{1}{2}} \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})}$$

and

$$E[\sqrt{S_n}] = 2^{\frac{1}{4}} \frac{\Gamma(\frac{2n+1}{4})}{\Gamma(\frac{n}{2})}.$$

The result depends on parameter n. Condition holds for e.g. $n = 10$.