# Stability analysys of retrial queing system with non Poisson input and constant retrial rate

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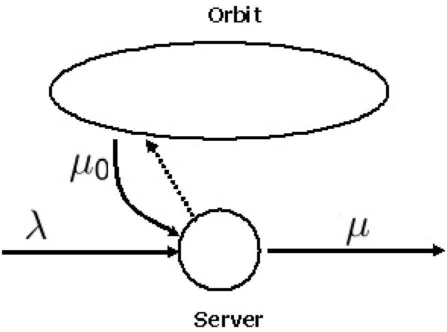
# Description of a model

Single-class retrial queueing system ($\Sigma$):




- Input of $\lambda$-customers
- General service time: $ES = 1/\mu$
- Infinite capacity orbit
- Poisson stream of orbit customers with rate $\mu_0$

# Motivation of the model

Applications:

- ALOHA type multiple access protocols [Choi, Rhee, Park(1993)]
- short TCP transfers [Avrachenkov, Yechiali(2008)]

**Classical systems:**

- $\tilde{\mu}_0 = \mu_0 \cdot n$, $n$ – number of orbit customers

**Considered system $\Sigma$:**

- $\mu_0 = \sum_{j=1}^{n} \frac{\mu_0}{n}$ – constant retrial rate

**Instability:** infinite growth of orbit size

# Stability criteria for $G/M/1/0$-type $\Sigma$

[Lillo (1996)]

$$\frac{\lambda(\mu+\mu_0)^2}{\mu\Big[\lambda\mu[1-B(\mu+\mu_0)]+\mu_0(\mu+\mu_0)\Big]}<1, \tag{1}$$

where

$$B(s) \;=\; \int_0^\infty e^{-xs}dF(x),\;\; F(x)=\mathrm{P}(S\le x). \tag{2}$$

# Sufficient stability condition

[Avrachenkov, Morozov (2010)] for $GI/G/m/n$-type $\Sigma$:

$$\mathrm{P}_{loss}(\lambda + \mu_0) < \mu_0 \tag{3}$$

Majorant **loss system** $\hat{\Sigma}$:

- Two independent input streams $(\lambda + \mu_0)$
- General service time as in $\Sigma$: $\mathrm{E}S = 1/\mu$
- Number of servers as in $\Sigma$
- Buffer size as in $\Sigma$

For M/G/1/0 case

$$\mathrm{P}_{loss} = \frac{\lambda + \mu_0}{\lambda + \mu_0 + \mu}. \tag{4}$$

# Regenerative estimation of $P_{loss}$ in $\hat{\Sigma}$

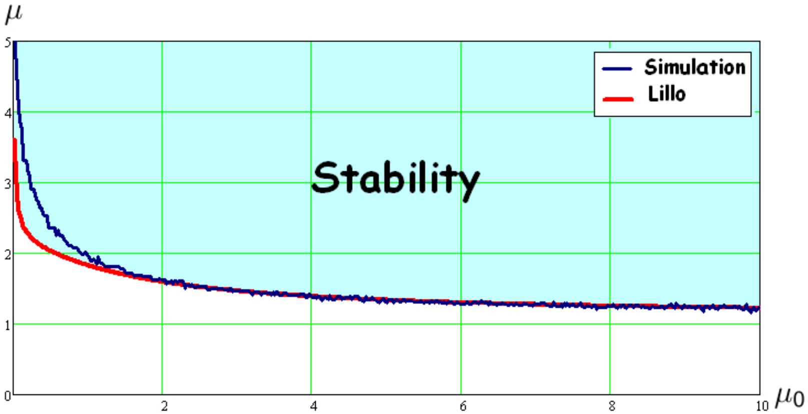
- $R(t)$, $A(t)$ – number of losses and number of arrivals in loss system $\hat{\Sigma}$
- $\nu(t)$ – queue length in $t$
- $\{t_n\}_{n\geq 0}$ – arrival instants
- $\beta_{n+1} = \inf_k\{k > \beta_n : \nu(t_k) = 0\}$, $n \geq 0$ – regenerative instants
- $R$, $A$ – generic number of losses, generic number of arrivals per cycle
- $\hat{P}_{loss}(t) := R(t)/A(t) \to \mathsf{E}R/\mathsf{E}A := P_{loss}$, w. p. 1.

## Alternative sufficient stability condition

$$\hat{P}_{loss}(t)(\lambda + \mu_0) < \mu_0. \quad (5)$$

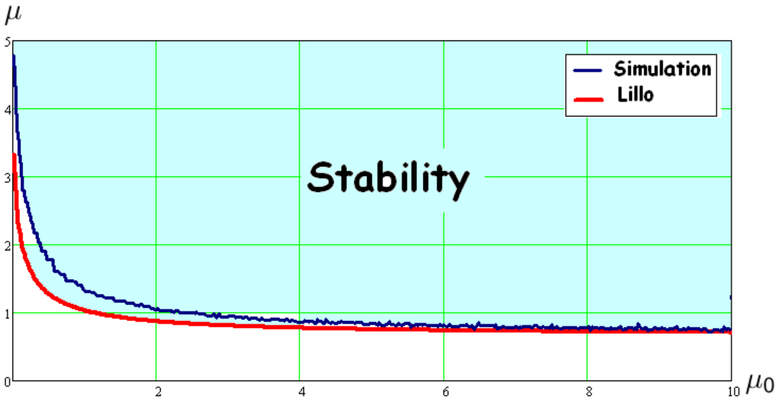
# Stability region

- Weibull distribution of input stream
- $P(\tau \leq x) = 1 - exp(-x^w), \;\; w := 4$
- $\lambda = 1/E\tau = (\int_0^\infty u^{1/w} e^{-u} du)^{-1}$
- exponentional service time



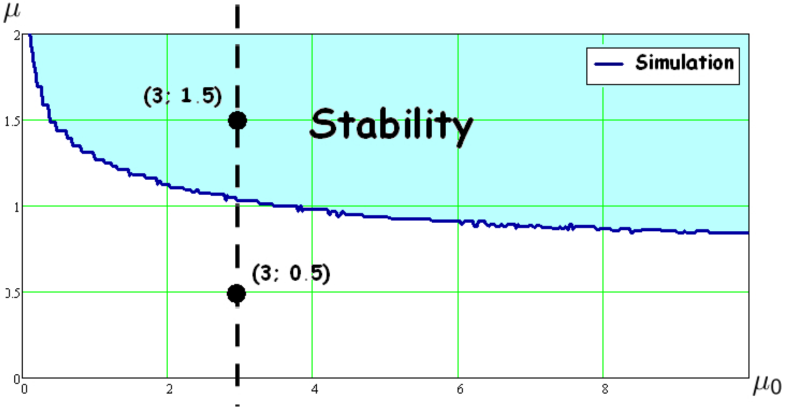
# Stability region

- Pareto distribution of input stream
- $\mathrm{P}(\tau \leq x) = 1 - x^{-\alpha}, \;\; \alpha := 3$
- exponentional service time



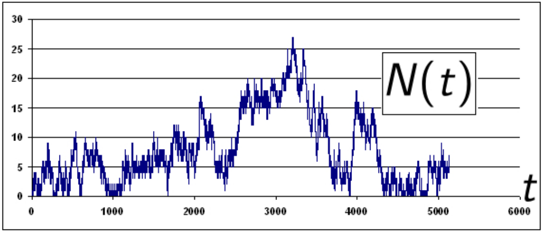
# Stability region

- Webull distribution of input stream, $w = 2$
- Deterministic service time with parameter $d$, $\mu = 1/d$

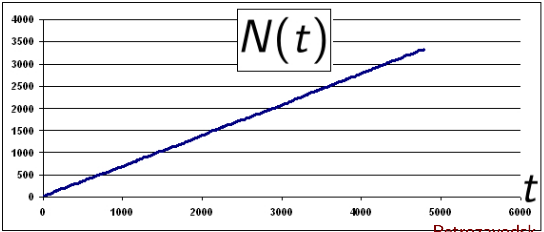


# Orbit dynamics $Webull/D/1/0$ case

$w = 2,\ \lambda = 1.128,\ d = 0.666,\ \mu_0 = 3$




$w = 2,\ \lambda = 1.128,\ d = 2,\ \mu_0 = 3$



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Thank you for your attention.