Simulation of the fluid system with long-range dependent input

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Gaussian traffic

Each source is described by ON/OFF process

$$\left\{ W^{(m)}(t), t \geq 0 \right\}$$, where $$W^{(m)}(t) = \begin{cases} 1, & t \in ON-period \\ 0, & t \in OFF-period \end{cases}$$

$$\overline{F_{ON}}(x) \sim x^{-\alpha_{ON}}, \quad 1 < \alpha_{ON} < 2$$

$$\overline{F_{OFF}}(x) \sim x^{-\alpha_{OFF}}, \quad 1 < \alpha_{OFF} < 2$$

Cumulative traffic of M sources on [0, tT]

$$W(tT) = \int_0^{tT} \sum_{m=1}^{M} W^{(m)}(u)\, du$$

Convergence to FBM

Interest is in the behavior of this process when M,T are large(Taqqu, 1997).

$$\lim_{T\to\infty}\lim_{M\to\infty}\left\{\frac{W(tT)-\dfrac{\mu_{ON}}{\mu_{ON}+\mu_{OFF}}TMt}{\sqrt{M}T^{H}}\right\}\overset{d}{=}c\,B_H(t),\ t\geq 0$$

here $$\frac{1}{2}\leq H=\frac{3-\min(\alpha_{ON},\alpha_{OFF})}{2}<1$$

It means that

$$W(tT)\approx\frac{\mu_{ON}}{\mu_{ON}+\mu_{OFF}}TMt+\sqrt{M}T^{H}cB_H(t)$$

System with finite buffer

Input: $$A(t) = mt + \sqrt{am}B_H(t)$$

Service: constant service rate C

Stationary overflow probability:

$$P(Q > b) = P\left(\sup_{t \geq 0}\left(A(t) - ct\right) > b\right)$$

System with finite buffer

$$Q(t) = Q_b(t-1) - c + m + \sqrt{am}\left(B_H(t) - B_H(t-1)\right)$$

$Q(t) < 0$

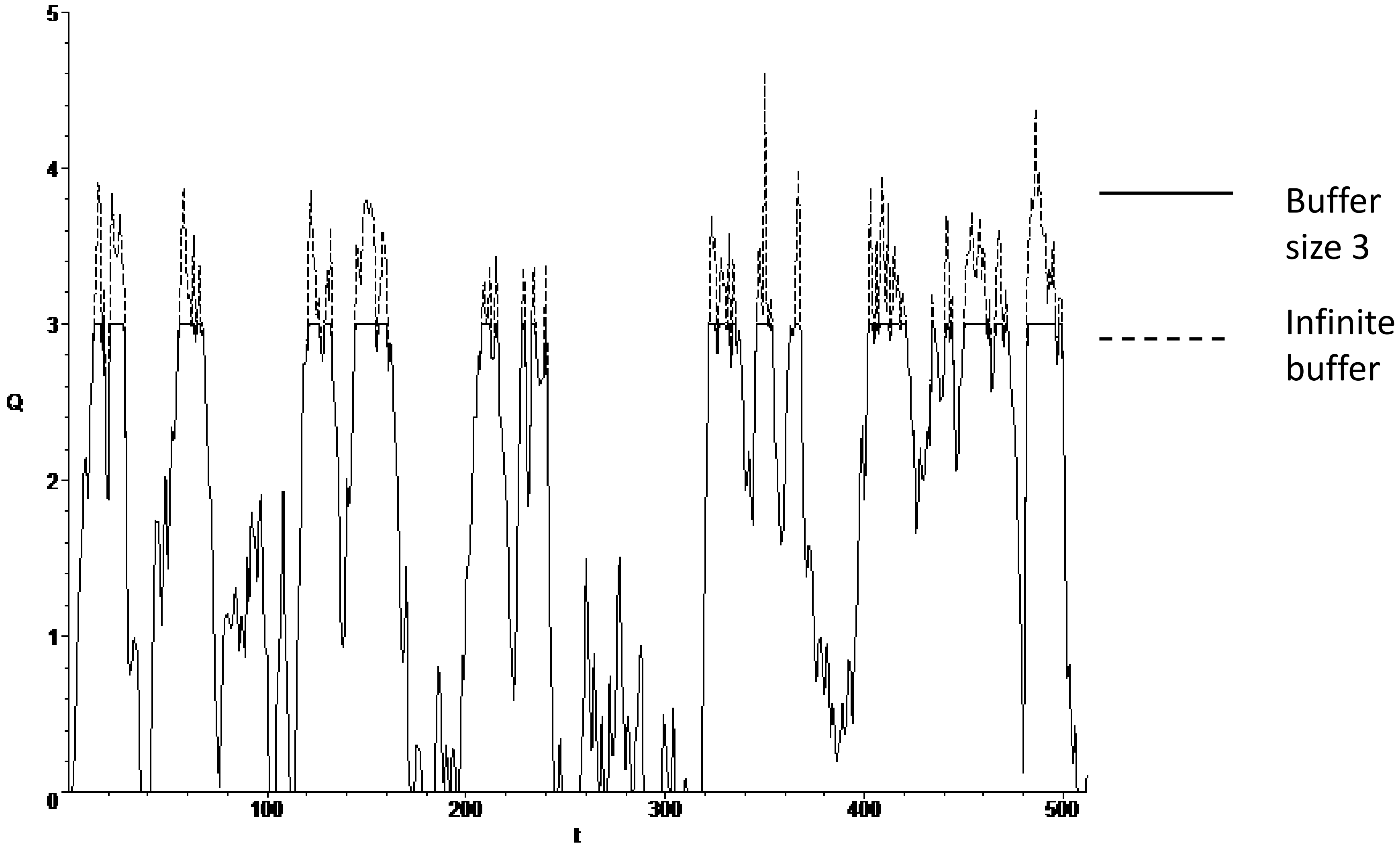
➡ $Q_b(t) = 0$

$Q(t) > b$

➡ $Q_b(t) = b$

$$Q_b(t) = \min\left(\left[Q_b(t-1) - C + m + \sqrt{am}\left(B_H(t) - B_H(t-1)\right)\right]^+, b\right)$$

System with finite buffer



Overflow and loss probability

Overflow probability (N – sample size):

$$P(Q > b) = \frac{\sum_{k=1}^{N} I\left(Q_k > b\right)}{N}$$

Loss probability on [0,T]:

$$P_{Loss}(b,T) = \frac{\sum_{k=1}^{T} \left[Q_b(t-1)+m+\sqrt{am}\left(B_H(t)-B_H(t-1)\right)-C-b\right]^{+}}{A(T)}$$

$$T \to \infty \qquad P_{Loss}(b) \approx \frac{E\left(Q_b+m+\sqrt{am}\left(B_H(t)-B_H(t-1)\right)-C-b\right)^{+}}{m}$$

[Kim & Shroff, 2001]

$$\frac{P_{Loss}(b)}{P(Q>b)} \approx const, \quad b \to \infty$$

Then

$$P_{Loss}(b) \approx \frac{P_{Loss}(0)}{P(Q>0)} P(Q>b)$$

Relative error

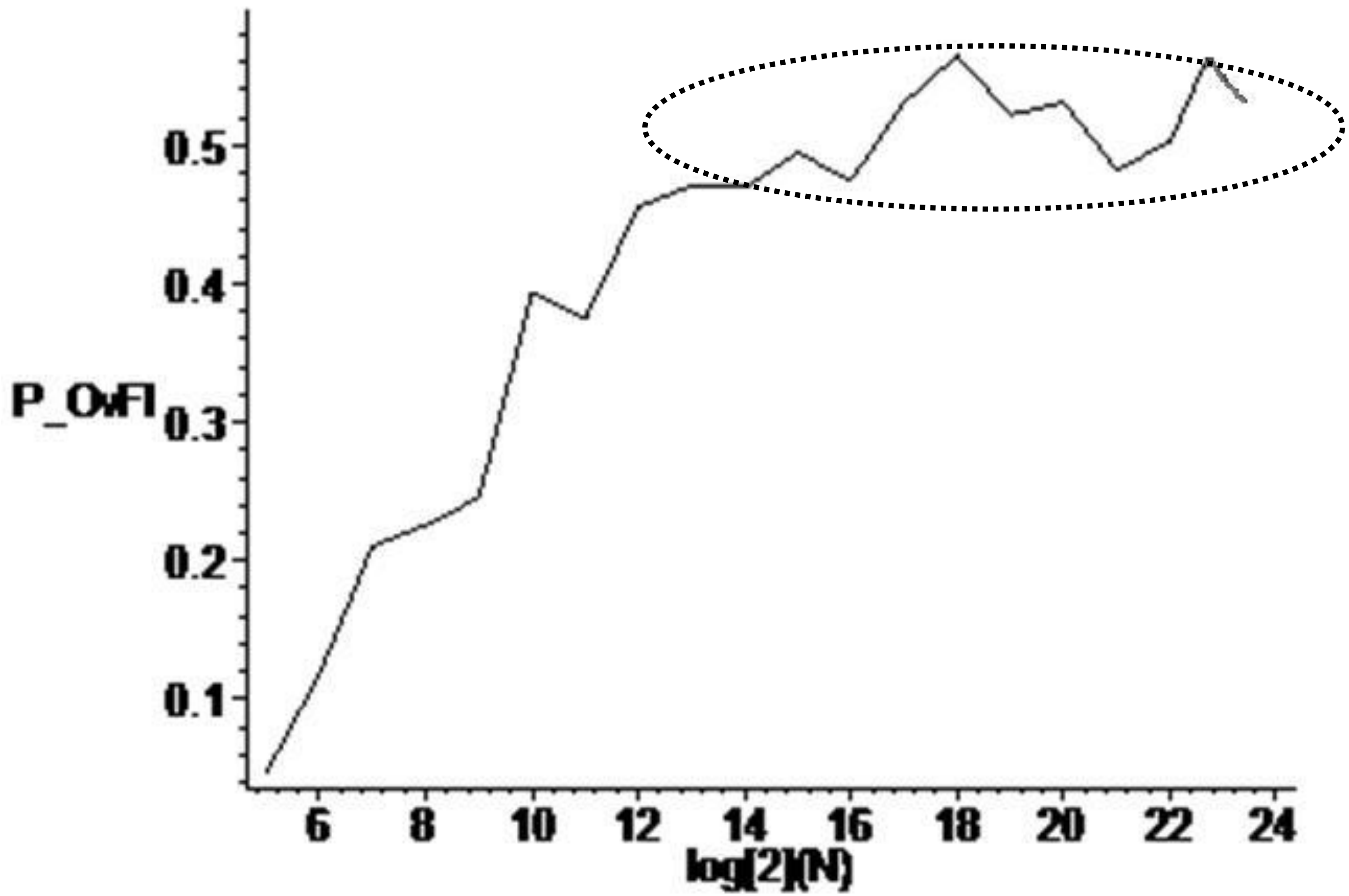
M – number of samples

$\tilde{p}_{Loss}$ - estimation of overflow probability

$$RE\left(\tilde{p}_{Loss}\right) = \frac{\sqrt{Var\left(\tilde{p}_{Loss}\right)}}{E\left(\tilde{p}_{Loss}\right)} \sim \frac{1}{\sqrt{p_{Loss}M}}, \quad p_{Loss} \to 0$$

It means that number of samples M must be sufficiently large.

Overflow probability – dependence on sample size



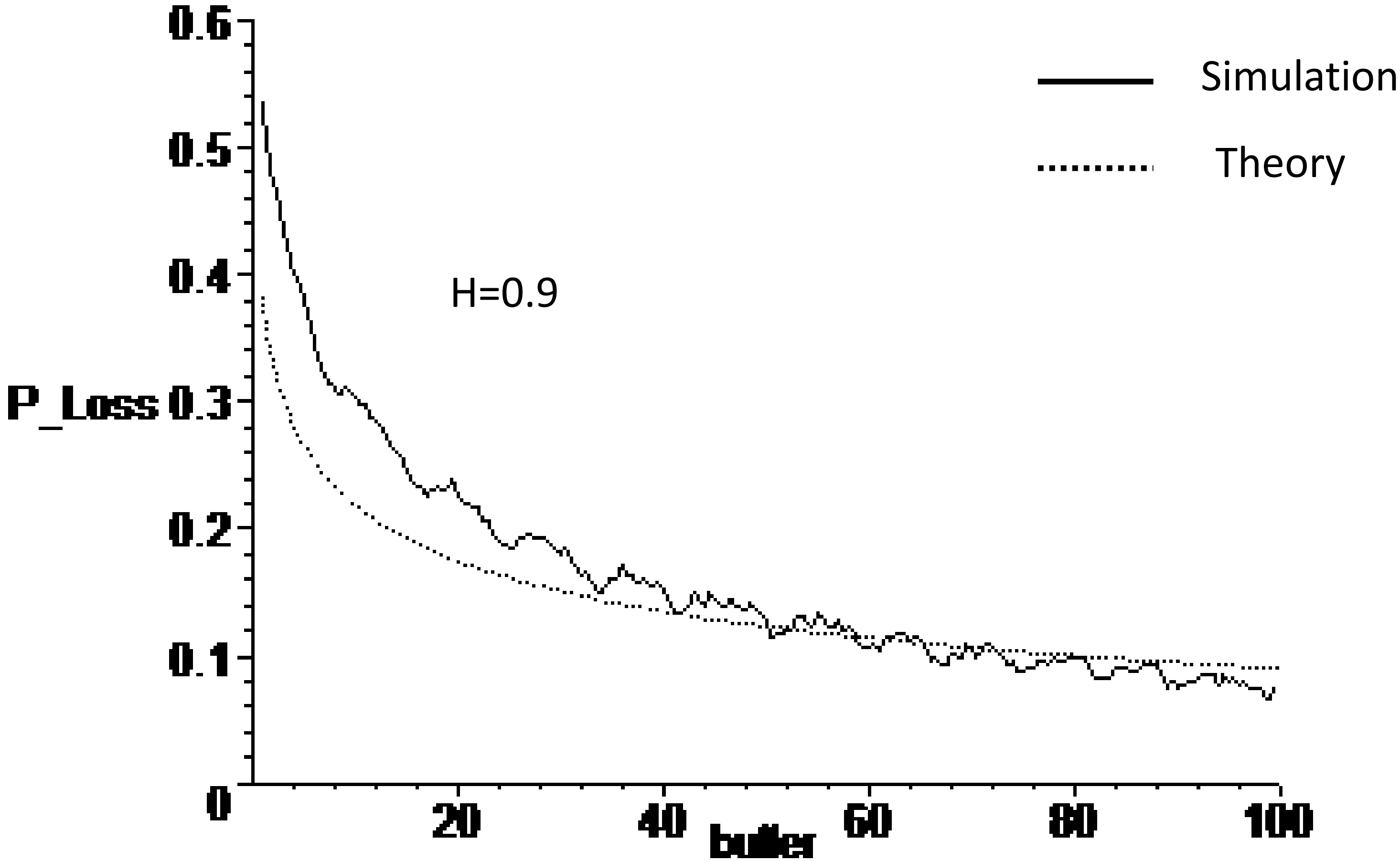
Overflow probability – theoretical results

[Duffield N., O'Connell N.,1995]

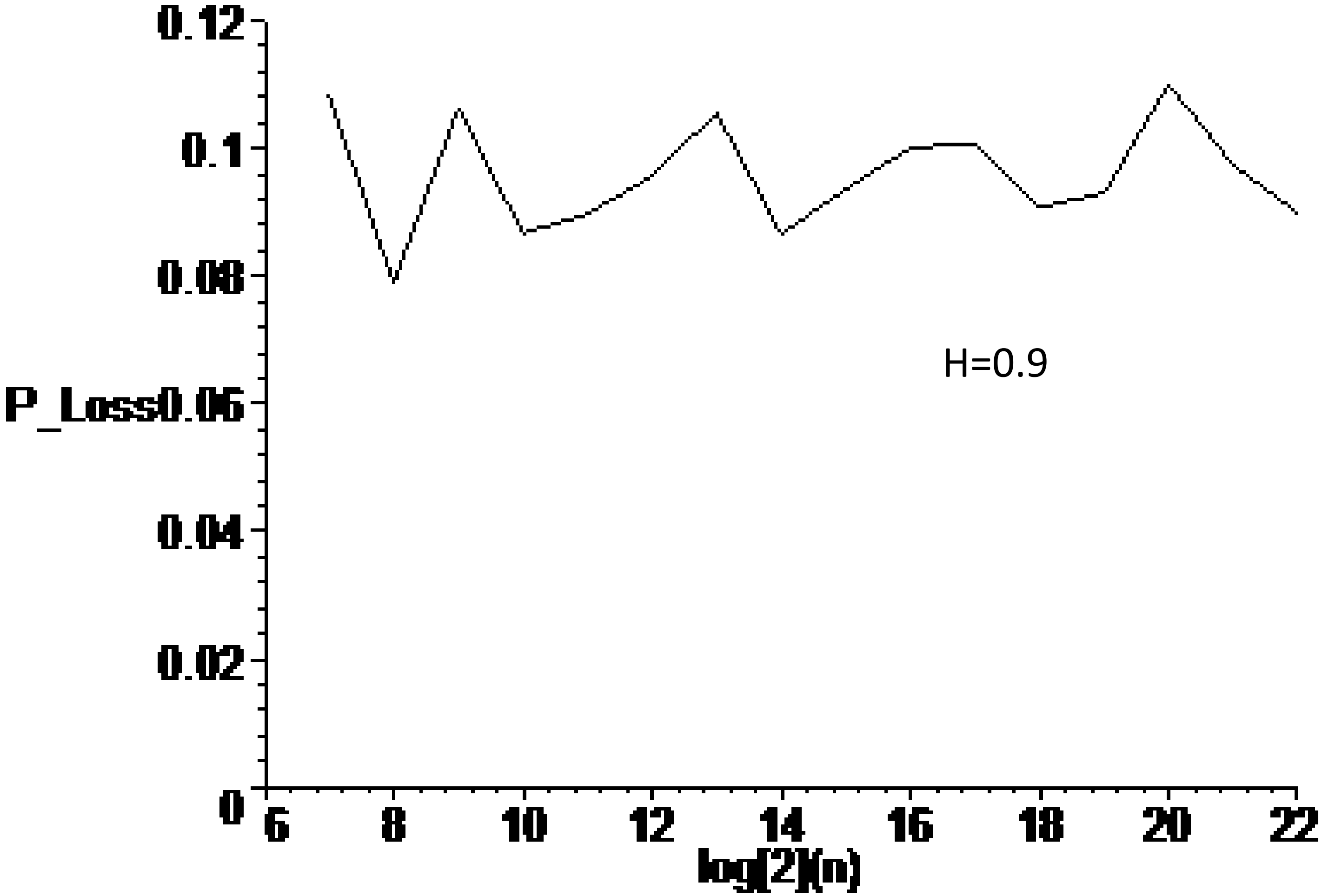
In the case of fractional Brownian motion input

$$P(Q > b) \approx \exp\left\{ -\frac{1}{2} \left(\frac{b}{1-H} \right)^{2-2H} \left(\frac{C}{H} \right)^{2H} \right\}$$

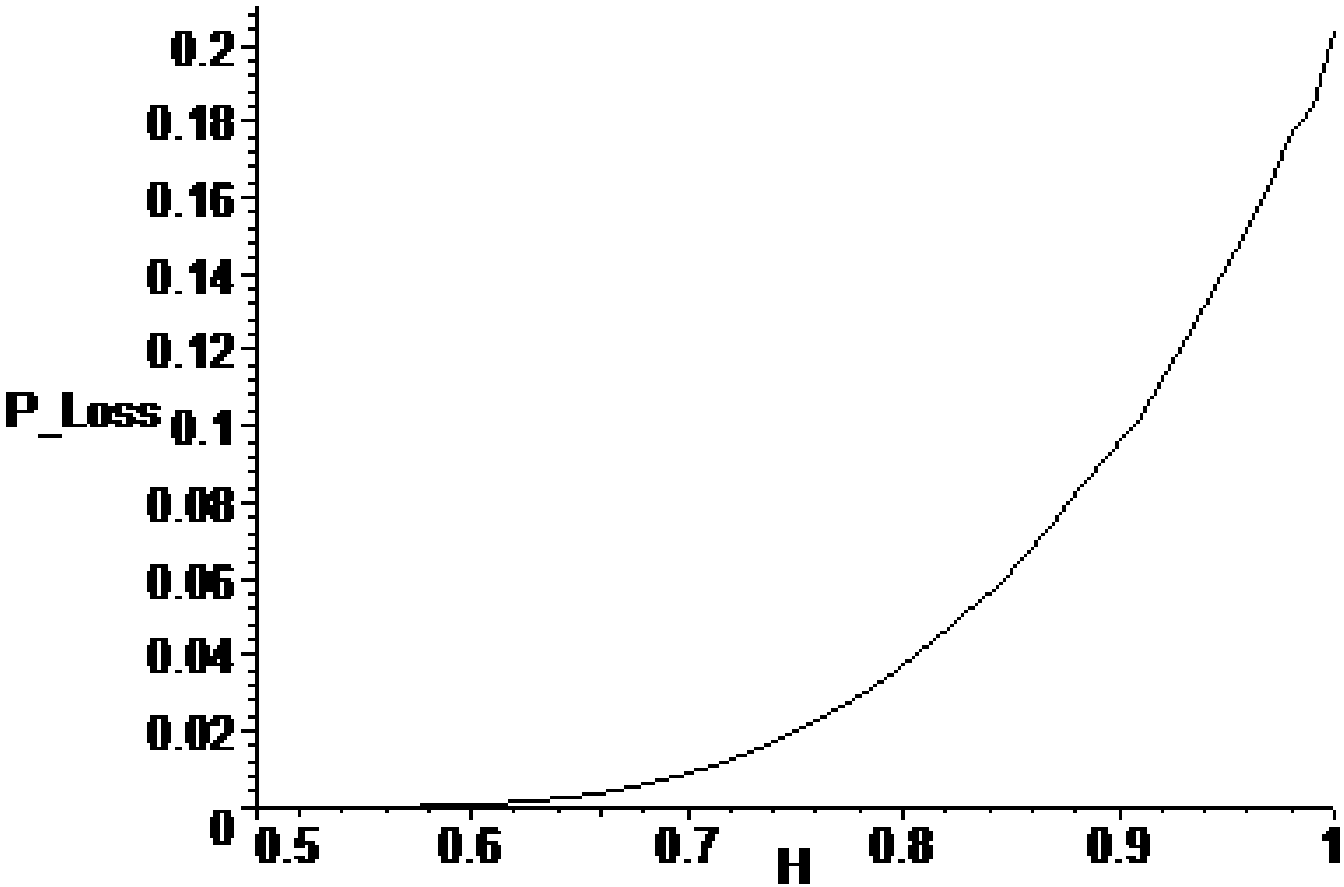
Overflow probability – comparison with theoretical results



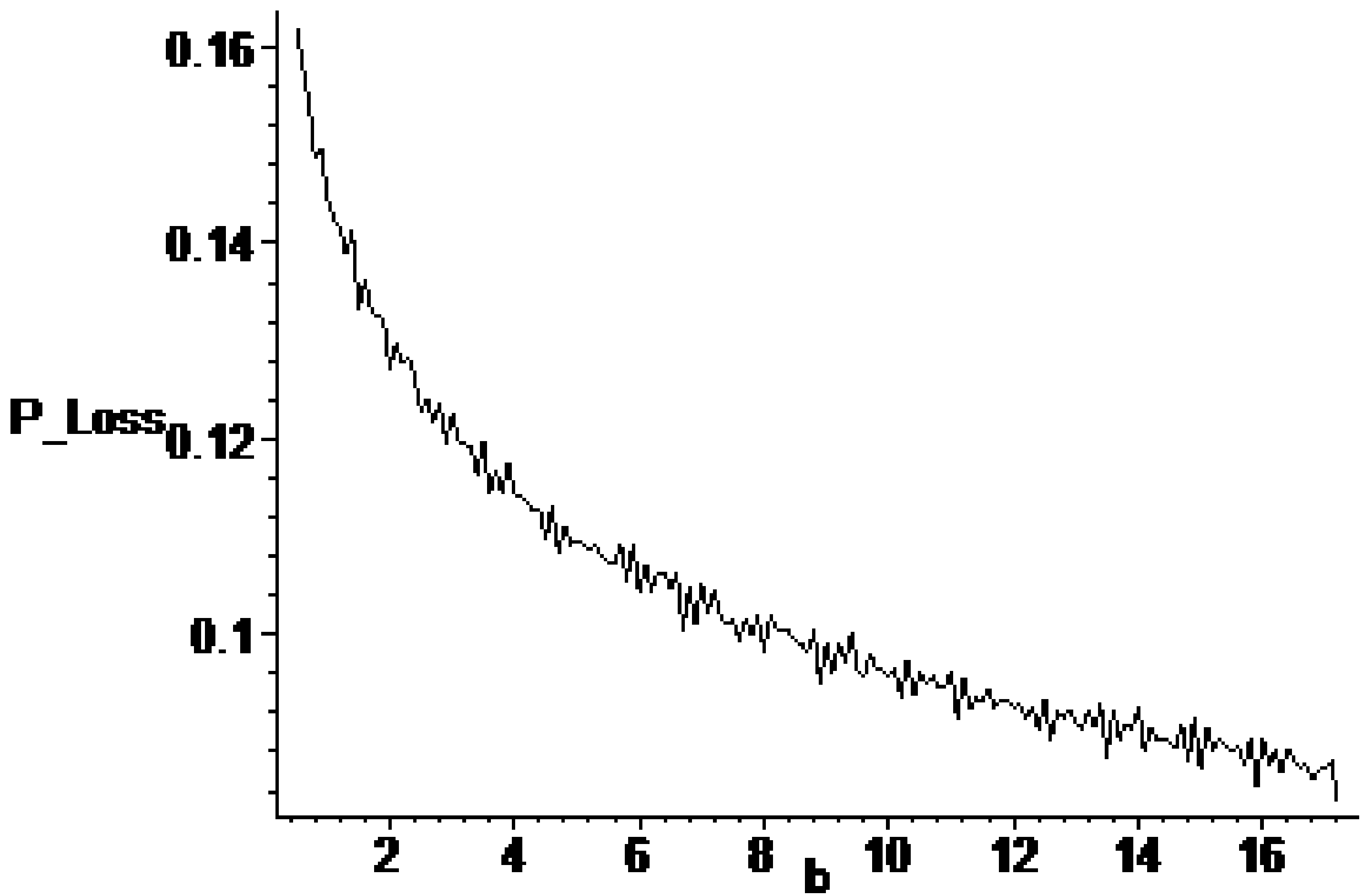
Loss probability – dependence on sample size



Loss probability – dependence on H



Loss probability – dependence on buffer size



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Thank you