Optimizing performance in heavy-tailed system: a case study

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Heavy-tailed distributions

Heavy-tails have been observed in:

- hydrology
- geology
- insurance
- risk analysis
- network analysis and computer science
- and others

Pareto distribution

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Pareto distribution

$$P(X > x) = x^{-\alpha}, \; x > 1, \; \alpha > 1$$

Some key properties:

- Pareto law (20 / 80)
- Infinite variance and (if $\alpha < 1$) infinite mean are possible
- Heavy tails can cause burstiness

Model and problem formulation

- M/G/1 system, service times S
- The tail distribution $\bar{B}(x) = P\{S > x\}$ – heavy tail
- The integrated tail distribution is $\bar{B}_r(x) = \frac{1}{ES} \int_0^x \bar{B}(y)dy$

How we can reduce a negative influence of heavy tails?

Practice recommendations:

1. Choose a service discipline
2. Choose a server architecture
3. Choose a task assignment policy

Case study 1: Choosing a service discipline

W – waiting time, V – sojourn time[1]

- First Come First Served

$$P\{W > x\} \sim \frac{\rho}{1-\rho}\bar{B}_r(x), \ x \to \infty$$

- Processor Sharing

$$P\{V > x\} \sim \bar{B}\big((1-\rho)x\big), \ x \to \infty$$

- Last Come First Served Preemptive-Resume

$$P\{V > x\} \sim \frac{1}{1-\rho}\bar{B}\big((1-\rho)x\big), \ x \to \infty$$

[1] The source is [1]

Case study 1: Choosing a service discipline

- Last Come First Served Non-Preemptive

$$P\{W > x\} \sim \rho \bar{B}_r\big((1-\rho)x\big),\ x \to \infty$$

- Foreground-Background Processor Sharing

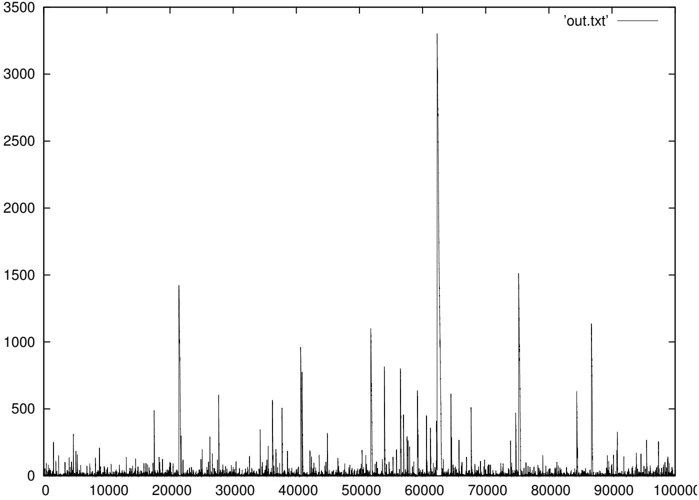
$$P\{V > x\} \sim \bar{B}\big((1-\rho)x\big),\ x \to \infty$$

- Shortest Remaining Process Time First

$$P\{V > x\} \sim \bar{B}\big((1-\rho)x\big),\ x \to \infty$$

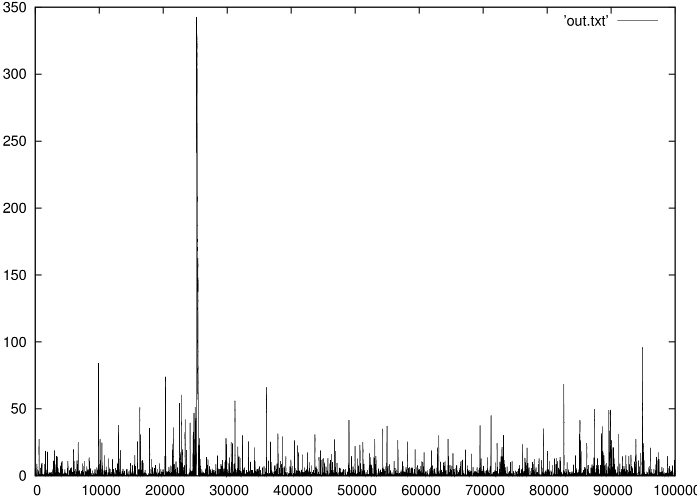
Case study 2: Choosing a server architecture

Simulating waiting time in M/G/1 system



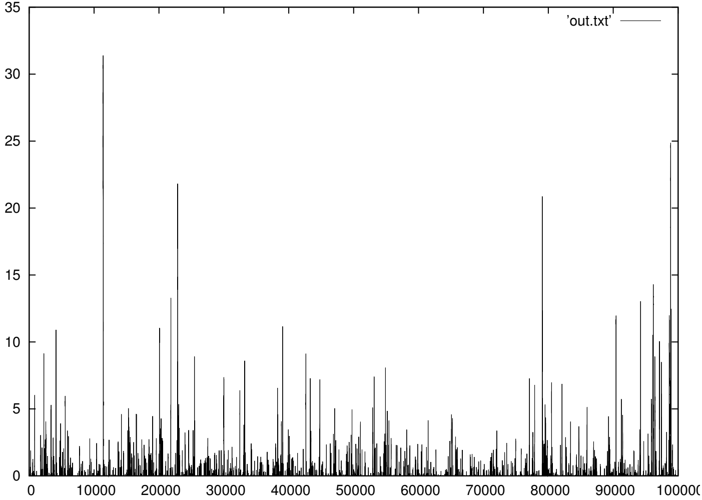
Case study 2: Choosing a server architecture

Simulating waiting time in M/G/2 system



Case study 2: Choosing a server architecture

Simulating waiting time in M/G/4 system



Case study 2: Choosing a server architecture

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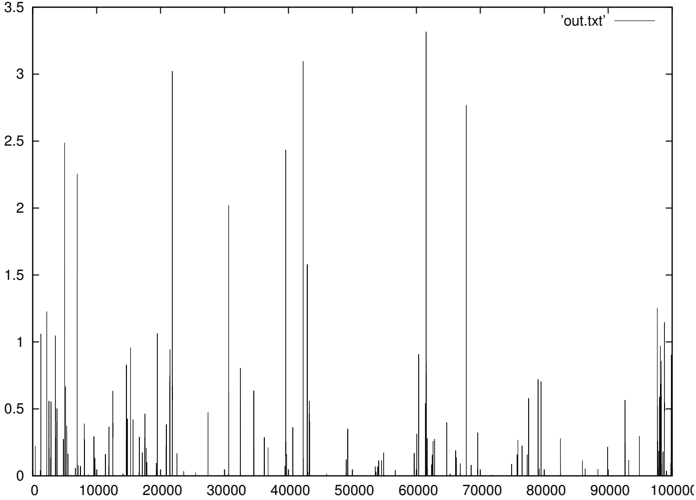
Simulating waiting time in M/G/8 system



Case study 3: Choosing a task assignment policy

M/G/n system, task sizes are bounded.

Bounded Pareto distribution $B(k, p, \alpha)$

$$f(x) = \frac{\alpha k^{\alpha}}{1 - (k/p)^{\alpha}} x^{-\alpha-1}, \; k \leq x \leq p$$

Task assignment policies:

- Random: a choice with equal probability
- Round-Robin: a cyclical order
- Dynamic: a core with the smallest amount of remaining work is selected
- Size-based: SITA-E defines the size range associated with each core

Case study 3: Choosing a task assignment policy

- SITA-E — Size Interval Task Assignment with Equal Load algorithm

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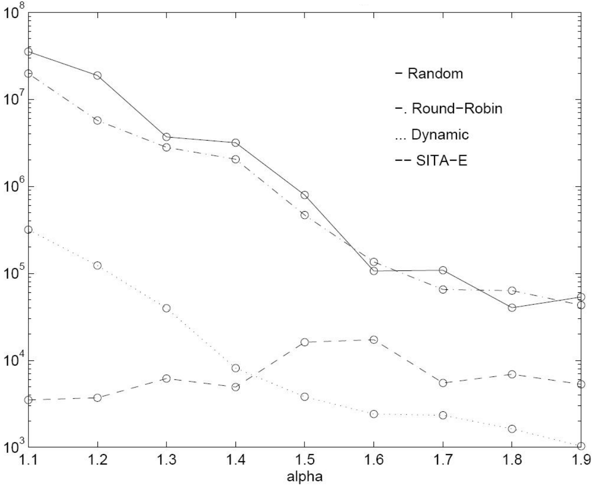
- The total work of each core is the same => mean waiting time decrease
- If $B(x)$ – the distribution function, M – mean tasks size; "Cutoff points" $x_i,\ i = 0..n,\ x_0 = k,\ x_n = p$ are defined by:

$$\int_{x_0}^{x_1} xdB(x) = \int_{x_1}^{x_2} xdB(x) = \ldots = \int_{x_{n-1}}^{x_n} xdB(x) = \frac{M}{n}$$

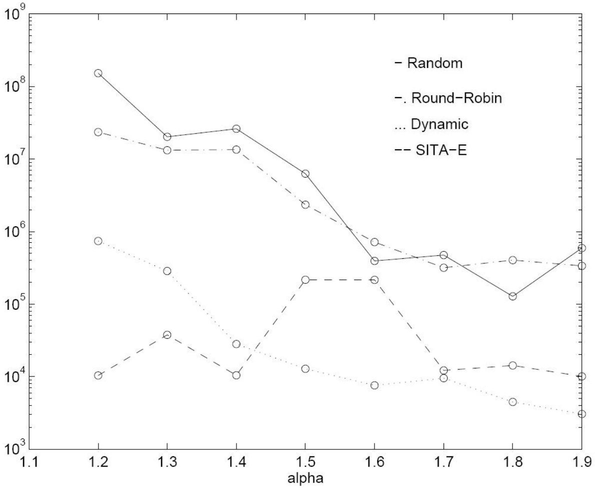
Simulated mean waiting time in M/G/n system[2]



[2]The source is [2]

Case study 3: Choosing a task assignment policy

Simulated standard deviation of waiting time in M/G/n system



References

- [1] Borst S.C., Boxma O.J., Nunez-Queija R. *Heavy Tails: The Effect of the Service Discipline*. 2002.
- [2] Harchol-Balter M. *The Effect of Heavy-Tailed Job Size Distributions on Computer System Design*. 1999.
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- [4] Samorodnitsky G. *Long Range Dependence, Heavy Tails and Rare Events*. 2002.
- [5] Zwart A. *Queueing Systems with Heavy Tails*. 2002.

Thank you for attention!