Some analytical aspects of regenerative simulation of fluid models

Ruslana Goricheva

Institute of Applied Mathematical Research Karelian Research Centre, RAS.
(joint work with Oleg Lukashenko, Dr. Evsey Morozov, Dr. Michele Pagano)

Fluid model

Input traffic:

- $A(t) = mt + \sqrt{m} \cdot X(t)$
- m - mean input rate
- $X(t)$ - random centred Gaussian process with

  covariance $\Gamma(t,s) = \mathbf{E}[X(t)X(s)]$

Service rate: Constant service rate *C*

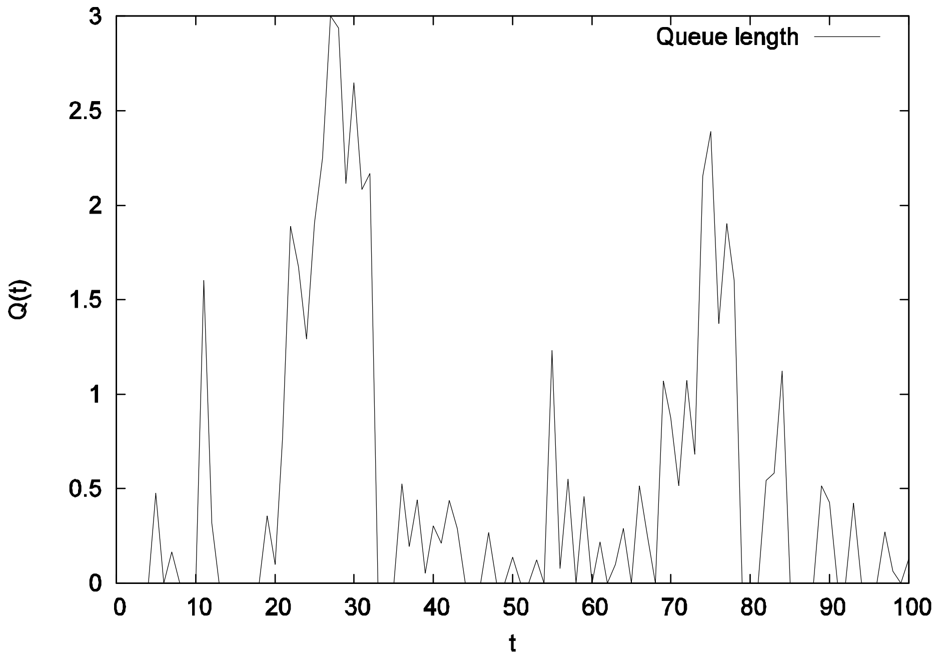
Queue with Brownian input

Input traffic:

$$A(t) = m\,t + \sqrt{m}\,B(t)$$

Queue length (discrete scale):

$$Q_b(t) = \min\left(\left(Q_b(t-1) - C + m + \sqrt{m}\left(B(t) - B(t-1)\right)\right)^{+}, b\right),\ \mathrm{t} = 0,1,\ldots$$



Regenerative approach

- Q_t - workload of a system at time t;
- EL - mean lost work per cycle;
- EA - mean workload arrived per cycle;
- $L_n(t)$ - lost work in [0; t] (n – buffer size);

Regeneration points:

$$\beta_{k+1} = \min\{t > \beta_k : Q_t = 0\}, \ \beta_0 = 0$$

By regeneration theory:

$$\lim_{t \to \infty} \frac{L_n(t)}{A(t)} = \frac{EL}{EA} \equiv P_l$$

Delta-method for confident estimation.

- X , Y- random variables;
- $z_1 := EX$, $z_2 := EY$;
- $Z := (z_1, z_2)$;

Confidence interval for $f(Z)$.

- $\hat{z}_i = \frac{\sum_{k=1}^{R} z_i^{(k)}}{R}$, $i = 1, 2$, $R \in N$;
- $\hat{Z} = (\hat{z}_1, \hat{z}_2)$;
- $f(\hat{Z})$ - point estimator of $f(Z)$.

By CLT:

$$\sqrt{R}\,(f(\hat{Z}) - f(Z)) \Rightarrow N(0, \sigma^2), \quad as \quad R \to \infty,$$

Where $\sigma^2 = \nabla f(Z)\,\Omega\,(\nabla f(Z))^T$ and Ω is a covariance matrix.

Ratio estimator.

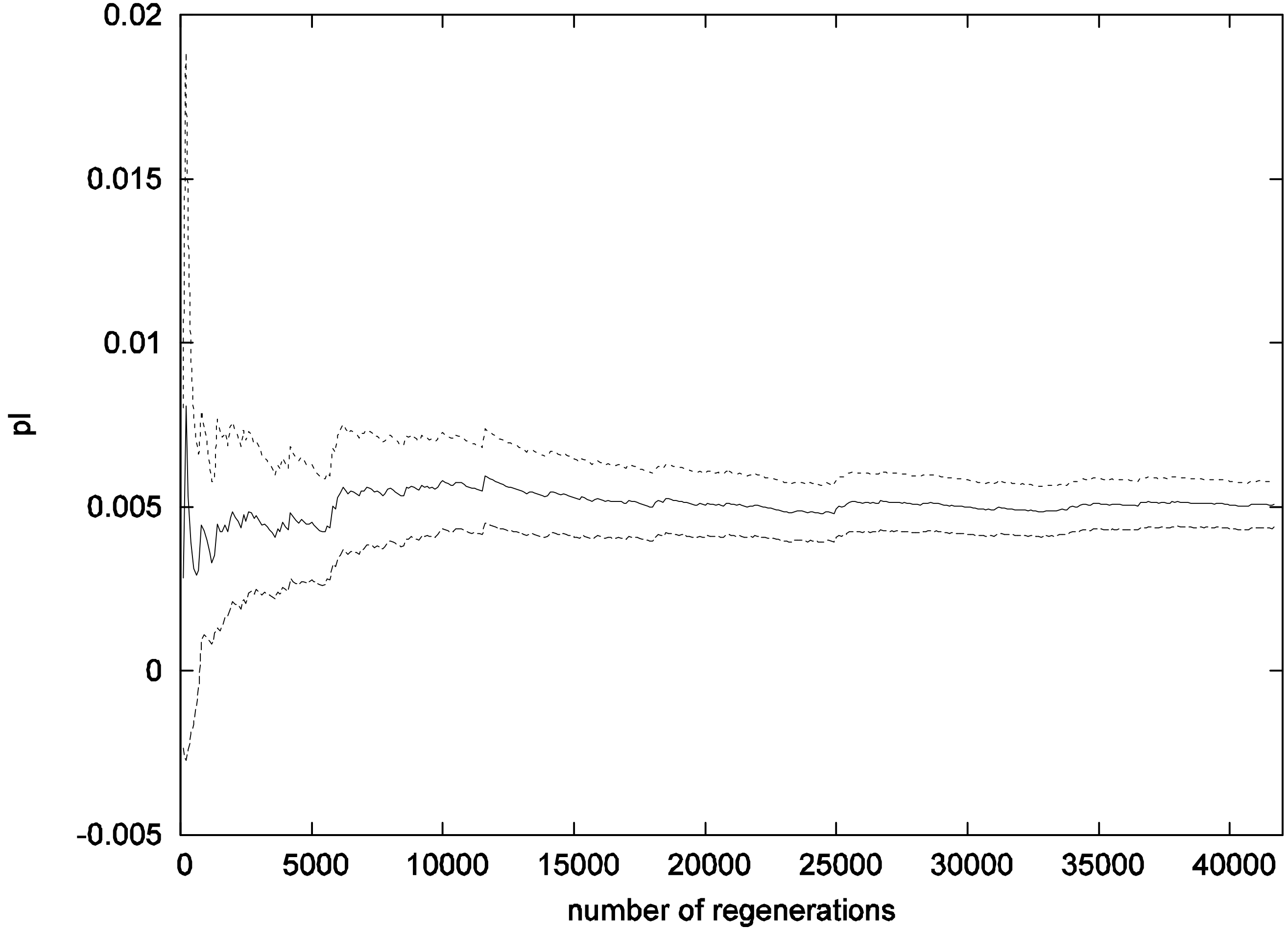
$$f(Z) = \frac{z_1}{z_2} = \frac{EL}{EA} = P_l.$$

Confidence interval for loss probability:

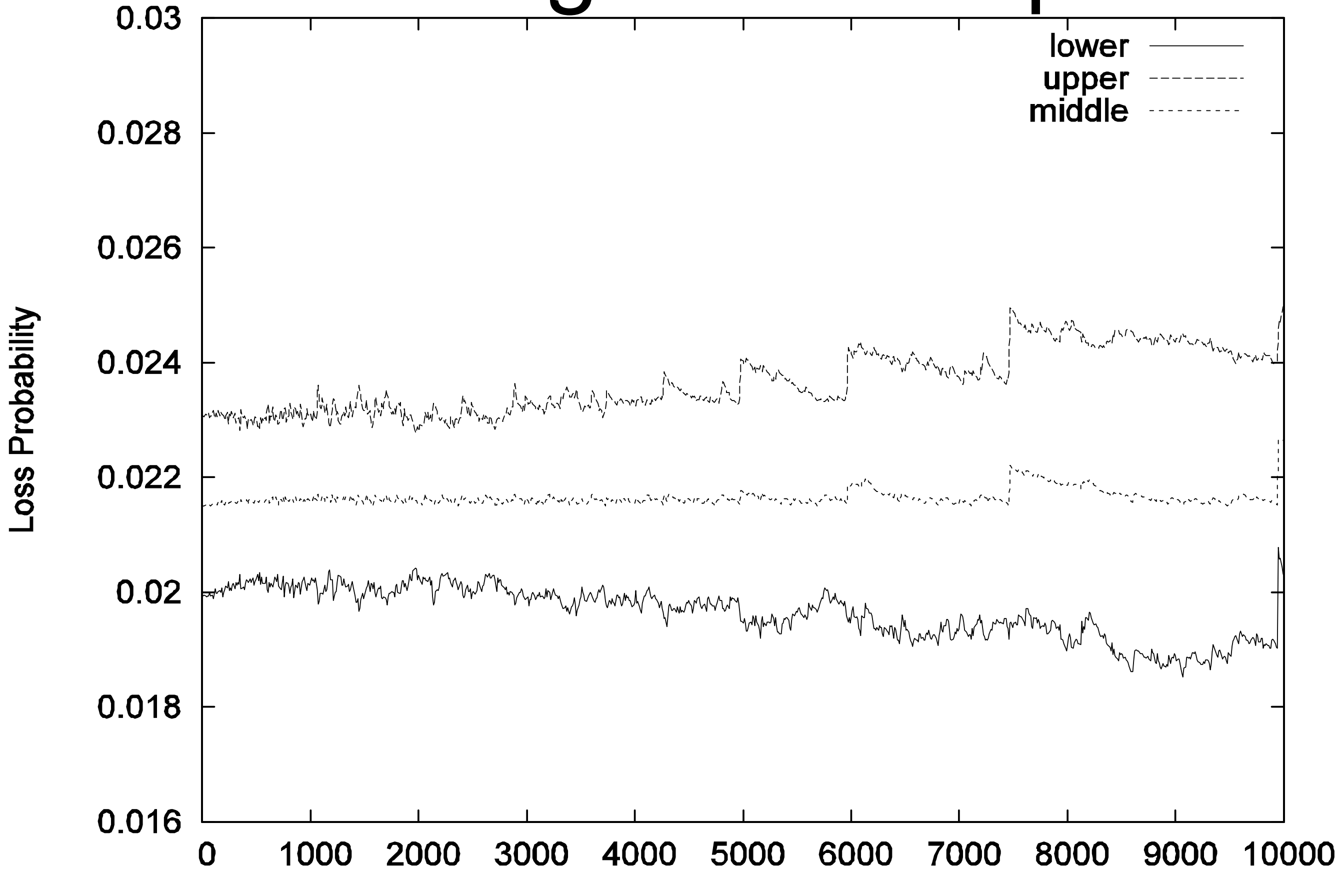
$$P_l \in [\hat{P}_l - \frac{z_\gamma}{\sqrt{R}}; \hat{P}_l + \frac{z_\gamma}{\sqrt{R}}],$$

where $z_\gamma = \sigma\Phi^{-1}(\frac{\gamma}{2}).$

Confidence interval for P_l in BM/D/1/n

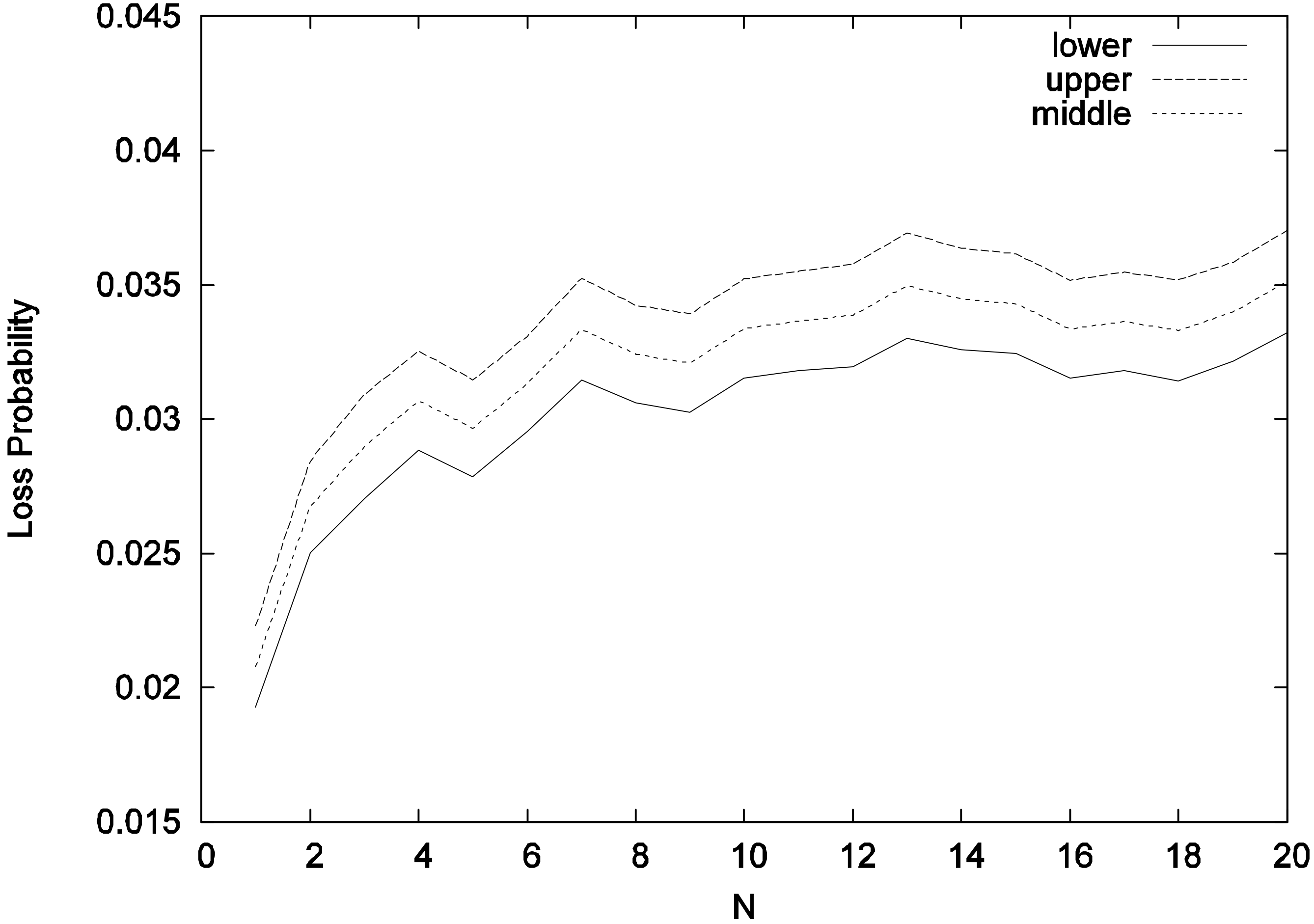


Confidence interval for different frequency of regeneration points



Discretization step

$$Q_b(t+h) = \min\left(\left(Q_b(t) - Ch + \left(A(t+h) - A(t)\right)\right)^{+}, b\right),\ \mathrm{h} \equiv \mathrm{h}_N = \frac{1}{N}.$$

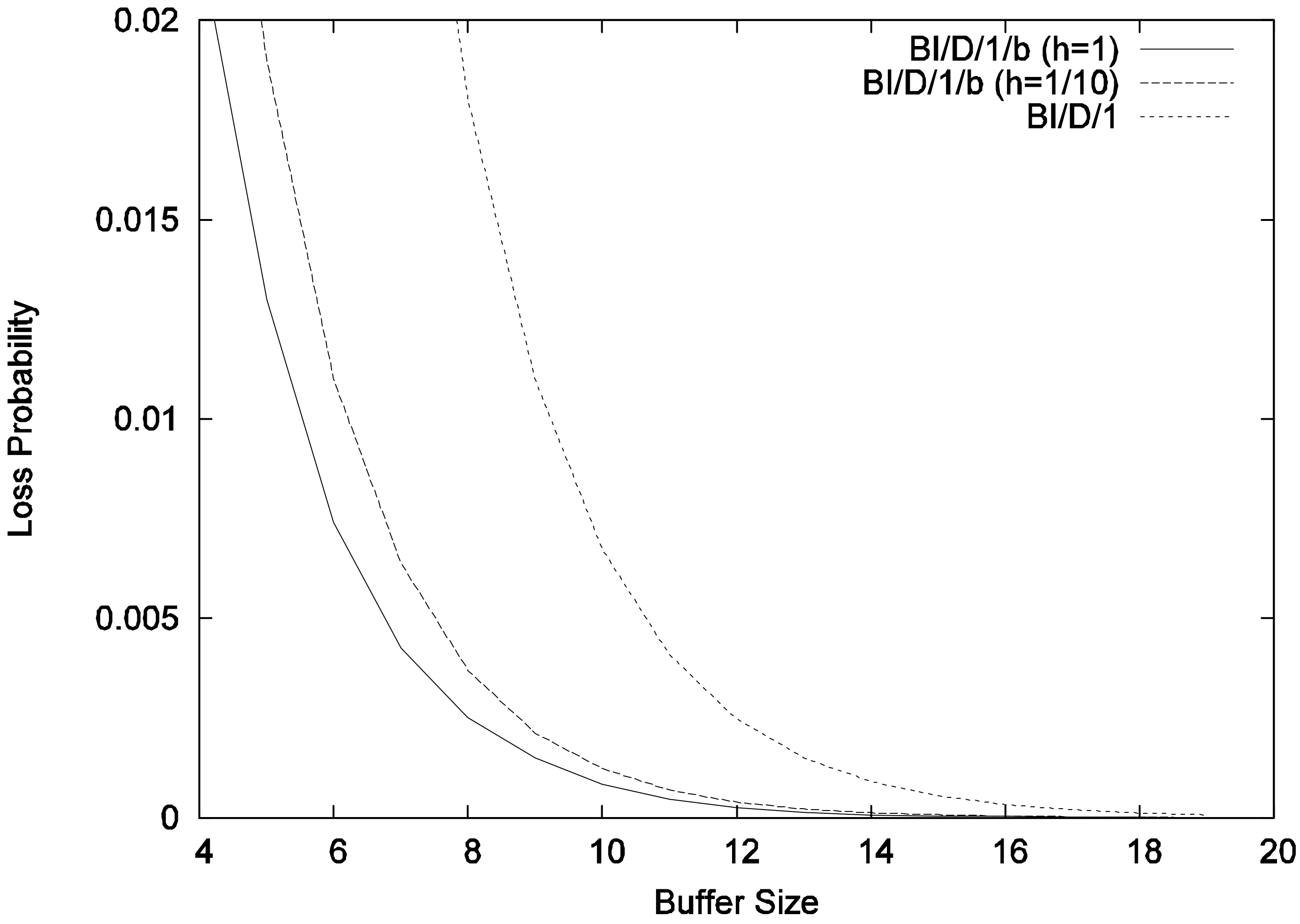


Comparison with BI/D/1

$$P(Q > b) = \exp\left(-2\frac{C-m}{m}b\right)$$ - overflow probability

Assumption:

$$\frac{P(Q > b)}{P_l(b)} \xrightarrow[b\to\infty]{} const.$$


BI/D/1/b (h=1)
BI/D/1/b (h=1/10)
BI/D/1
Loss Probability
0.02
0.015
0.01
0.005
0
4
6
8
10
12
14
16
18
20
Buffer Size

Simulation of BM

$$B\left(t_n^h\right) = \Delta_1^h B + \cdots + \Delta_k^h B, \quad k = 1, ..., n,$$

where

$$\Delta_n^h B \sim N(0, h) \qquad h = h_N = 1 / N,$$

Error of linear interpolation:

$$\mathbf{E}\int_0^1 \left| B^h(t) - B(t) \right| dt = c / N^{1/2}, \quad c = \sqrt{\pi / 32}$$

Conclusions

- Analysis of queue with Brownian input via regeneration method, based on i.i.d. property of increments of BM.

- Confidence estimation of loss probability via delta-method.

- Estimation for different frequency of regeneration points.

- Analysis of value of discretization step.

- Comparison of observed results with explicit expression on continuous time for BI/D/1 .

References

- Asmussen S. Applied probability and Queues. Springer, 2003.
- Asmussen S., Glynn P. Stochastic Simulation: algorithms and analysis. Springer, 2007.
- *Law A. M., Kelton W. D.* Simulation modeling and analysis/ New York: McGraw- Hill, 1991. 2nd edt.
- Mandjes M. Large Deviations of Gaussian Queues. - Chichester: Wiley, 2007.
- Norros I. A storage model with self-similar input, Queueing Systems, vol. 16, pp. 387-396, 1994.

Thank you.