Simulation of the fluid system with long-range dependent input

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Abstract

We discuss the application of the simulation to estimate the loss or overflow probability in a queuing system with a finite or infinite buffer, which is fed by a Gaussian input. We mainly consider fractional Brownian input (FBI) because it satisfies some properties such as self-similarity and long-range dependence that network traffic sometimes obeys. This work is supported by Russian Foundation for Basic research, project No 10-07-00017 and done in the framework of the Strategy development Program for 2012-2016 "PetrSU complex in the research-educational space of European North: strategy of the innovation development.

1 Introduction

We consider the so-called fluid queue with a constant service rate and a Gaussian input process. The work focuses on the estimation of the overflow probability $P(Q > b)$, that is the probability that the workload process exceed a threshold level b (in the infinite buffer case) and the loss probability P_ℓ, or the buffer overflow probability (in the finite buffer queue). Such probabilities can be useful for the QoS analysis of telecommunication systems. At the present time, for the queues with general Gaussian input (in particular, for the most important models with fractional Brownian input (FBI)) there are no explicit results, and only some asymptotics for the overflow probability are found. Thus, in general, only simulation remains an available and the most adequate way to estimate the required probability.

Finite buffer systems, being more realistic models of real-life networks, are more difficult to be analyzed, and by this reason explicit (and asymptotic) expressions for P_ℓ in such systems are much less available.

2 Queue with long-range dependent input

In this section we describe a single server queue with deterministic service rate C. Denote by $A(t)$ the amount of data (input traffic) arrived into a communication node within time interval $[0, t]$, $t \geq 0$. Let consider the following definition of the input [4]:

$$A(t) = mt + \sigma B_H(t)$$

where $\{B_H(t),\, t \geq 0,\}$ is a fractional Brownian motion (fBm), which describes random fluctuations of the input around its linearly increasing mean, $H \in (1/2,\, 1)$, $\sigma > 0$ is some scaling parameter. Let $r = C - m$, to guarantee stability of such a system we assume that $r > 0$.

Firstly consider system with infinite buffer (FBI/D/1). Denote by $B^*_H(n) = B_H(n+1) - B_H(n)$ the increments of fBm. Accorging to Lindley recursion (in discrete time) we have following expression for workload (queue content):

$$Q(t) = \left(Q(t-1) - r + \sigma B^*_H(t)\right)^+ ,\ t = 1, 2, \ldots \tag{2.1}$$

where $(x)^+ = \max(0, x)$. The overflow probability (queue tail probability) is defined as the amount of time the queue spends above some level b divided by the total time:

$$\mathsf{P}(Q > b) = \lim_{T\to\infty} \frac{1}{T} \sum_{k=1}^{T} I(Q(k) > b), \tag{2.2}$$

where I means indicator. Recursion (2.1) can be extended to the queue content $Q_b(t)$ in a system with buffer size $b < \infty$ (FBI/D/1/b) as follows:

$$Q_b(t) = \min\left((Q_b(t-1) - r + \sigma B^*_H(t))^+, b\right),\ t = 1, 2, ... \tag{2.3}$$

The time average loss $\mathsf{P}_\ell(b, T)$ in this system during (discrete-time) interval $[0, T]$, is naturally defined as the ratio of the amount of loss to the total amount of input during this interval.

$$\mathsf{P}_\ell(b, T) = \frac{\sum_{k=1}^{T} (Q_b(k-1) - r + \sigma B^*_H(k) - b)^+}{A(T)}.$$

Under stability assumption, one can expect that stationary loss ratio converges to stationary loss probability $\mathsf{P}_\ell(b)$, that is

$$\mathsf{P}_\ell(b) = \lim_{T\to\infty} \mathsf{P}_\ell(b, T) = \frac{\mathsf{E}(Q_{n-1} + \sigma B^*_H(k) - r - b)^+}{m}. \tag{2.4}$$

Unfortunatelly, so far there are no explicit results for (2.2), (2.4) in case FBI. But there are some asymptotic results. For example, for sufficiently large b there is continious-time approximation

$$\mathsf{P}(Q > b) \approx \exp\left(-\theta b^{2-2H}\right), \tag{2.5}$$

where constant θ depends on initial parameters as:

$$\theta = \frac{r^{2H}}{2\sigma^2} \cdot \frac{1}{H^{2H}(1-H)^{2(1-H)}}.$$

Actually, expression (2.5) means that for sufficiently large b queue tail distribution $\mathsf{P}(Q > b)$ is Weibullian.

Kim and Shroff have established the relationship between overflow and loss probability [2]:

$$\log \mathsf{P}(Q > b) - \log \mathsf{P}_\ell(b) = O(\log b), \;\; as \;\; b \to \infty. \tag{2.6}$$

Let rewrite (2.6) as

$$\mathsf{P}_\ell(b) = \mathsf{P}(Q > b) b^{O(1)}, \; b \to \infty \tag{2.7}$$

Using the last expression, it is possible to derive approximation for the loss probability $\mathsf{P}_\ell(b)$ via the corresponding overflow probability. But existence of unknown function $O(1)$ in (2.7) makes it difficult in practice.

Let conider the boundary case $H = 1$. Obviously, $B_1(t) = t \cdot N(0,1)$ is a random line (the increments are the same). So there are two alternatives: first, when the system can complite all work and there are no losses; second, when the system can not complete work and after several steps the buffer is full and all subsequent work will be lost. It follows from above given arguments that there is explicit formula for $\mathsf{P}_\ell(b)$:

$$\begin{aligned} \mathsf{P}_\ell(b) &= \frac{\mathsf{E}(N(0,\sigma^2) - r)^+}{m} \\ &= \frac{1}{m\sigma\sqrt{2\pi}} \int_r^\infty (x-r) e^{-x^2/2\sigma^2} dx. \end{aligned} \tag{2.8}$$

3 Simulation

However, all given above results are asymptotic. So simulation is often the only way to calculate the overflow/loss probability for small or moderate values of b.

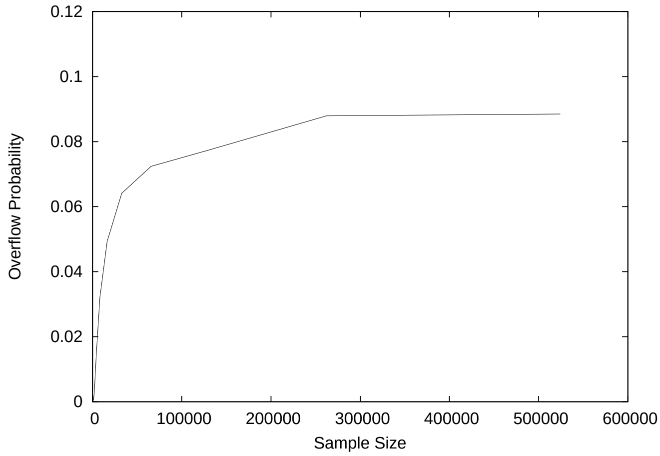

Figure 1: Stabilization of $\widehat{\mathsf{P}}_{500}$

let describe in brief the procedure of simulation. Firstly we should generate sufficiently large number N of fBm traces (sample paths). For generating fBm traces we use RMD-method (Random Midpoint Displacement) [3]. Each sample path should include sufficienly large number of observations T. After that based on (2.1), (2.3) we calculate sample paths $\{\widehat{Q}^i(k),\ \ k=1,...,T\}$, $\{\widehat{Q}^i_b(k),\ \ k=1,...,T\}$, $i=1,...,N$. Estimate for $P(Q>b)$ has following form:

$$\widehat{\mathsf{P}}_b = \frac{1}{T}\sum_{k=1}^{T} I\left(\widehat{Q}(k) > b\right).$$

Let L_n be the total amount of losses for the sample path $\widehat{Q}^n_b$. Then the estimate of $\mathsf{P}_\ell(b)$ is defined as folllows:

$$\widehat{\mathsf{P}}_\ell(b) \approx \frac{\mathsf{E}L}{mT}, \quad \mathsf{E}L = \frac{1}{N}\sum_{n=1}^{N} L_n$$

Figure 2 shows the dependence of the estimate of the overflow probability on sample path size T. Note that the sample path size must be sufficiently large to eliminate the influence of so-called initial period. The following parameters are used: $C=1$; $m=0.8$; $b=500$; $N=1000$.

Figure 2 compares the simulation results for the overflow probability in system $FBI/D/1$ with the approximation (2.5). The following parameters are used: $C=1$; $m=0.8$; $T=2^{16}$, $N=1000$. As expected from the

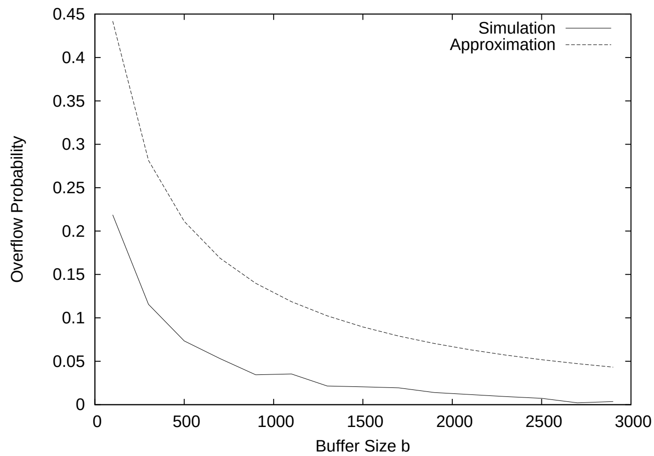

Figure 2: Simulation vs. approximation (2.5)

theory the difference between the simulation results and approximation decreases when the buffer size grows.

Finally, figure 3 shows the simulation results of P_ℓ for the finite buffer system $FBi/D/1/b$ with parameter $H = 0.99$, According to (2.5), for H close to 1 the loss probability it is easy to calculate approximate value of P_ℓ. For given parameters $C = 1$; $m = 0.8$ we have $\mathsf{P}_\ell \approx 0.297$ and it does not depend on buffer size b.

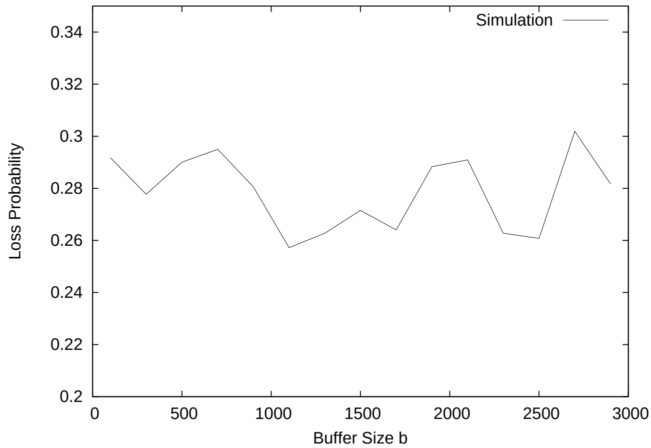

Figure 3: Estimate of P_ℓ for $H = 0.99$

4 Conclusion

The results of our study can be summarised as follows:

The procedure of estimation of the overflow and loss probability is discussed. We have compared the known approximation and the estimation using simulation technique. Numerical results are consistent with theoretical formulas.

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