Moment properties and long-range dependence of queueing processes

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Queues with Heavy Tail service

- Consider a single server GI/G/1 queue. D-stationary delay, W-waiting time, S-service
- Kiefer, Wolfowitz, 1956. Under stability cond., **finite ED^n <=> finite ES^{n+1}**
- Daley, 1968. GI/G/1, ES^3 finite, ES^4 infinite => **divergence of sum of corr(W_0,W_n), LRD**
- Morozov, 2009. ES^3 finite => **finite variance of unfinished regeneration time**, use regenerative approach

Extensions of the idea

- [Sigman, Huang, 1999] G/G/1 → /G/1 tandem queues
- [Scheller-Wolf, Sigman, 1996; Scheller-Wolf, 1999; Scheller-Wolf, Vesilo, 2006] G/G/s FIFO queues
- Their aim: derive moment asymptotics for stationary W and D under certain special conditions (e.g. heavy-tailness) to extend the main result.
- Bodyonov, Morozov, 2004. Regenerative simulation of a tandem network with LRD workload process. FDPW'2004.

Tandem queue

- $N \geq 1$ nodes connected in a sequence
- M/G/1 → /G/1 → … → /G/1
- Exp(λ) inter-arrival distribution (1 node)
- ? inter-arrival distribution (2:N nodes)
- Equal pareto(α) service time distributions

- Waiting time on the K-th node?

Lindley recursion

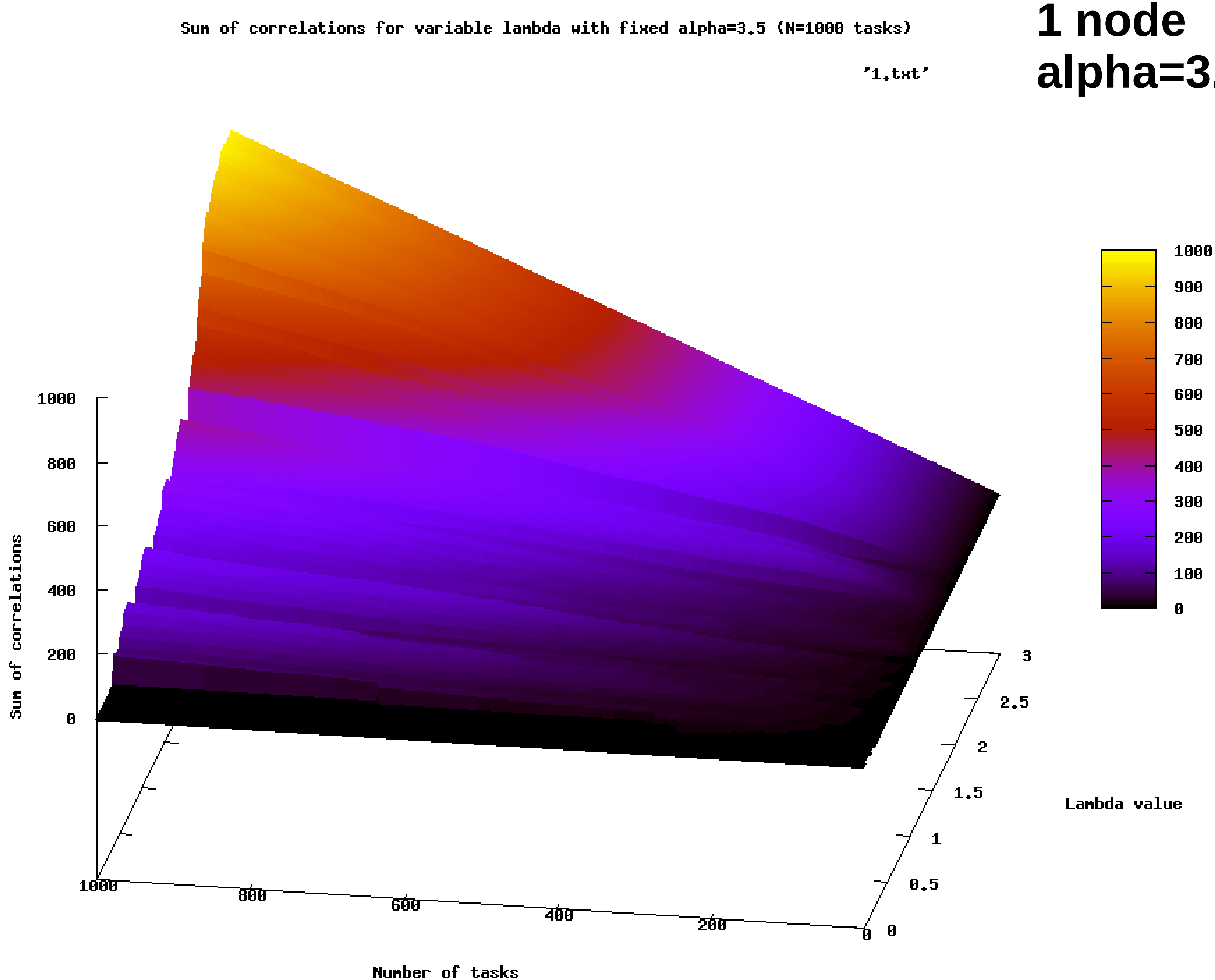
- $W_1(n+1)=(W_1(n)+S_1(n)-T_1(n))^+$
- $T_K(n)=(T_{K-1}(n)-W_{K-1}(n)-S_{K-1}(n))^+ + S_{K-1}(n+1)$
- Hence, $W_K(n+1)=(W_K(n)+S_K(n)-T_K(n))^+$
- $Cov(W(0),W(n))=(\frac{1}{N}\Sigma W^i(0)W^i(n)-\frac{1}{N}\Sigma W^i(0)\frac{1}{N}\Sigma W^i(n))$

Instruments

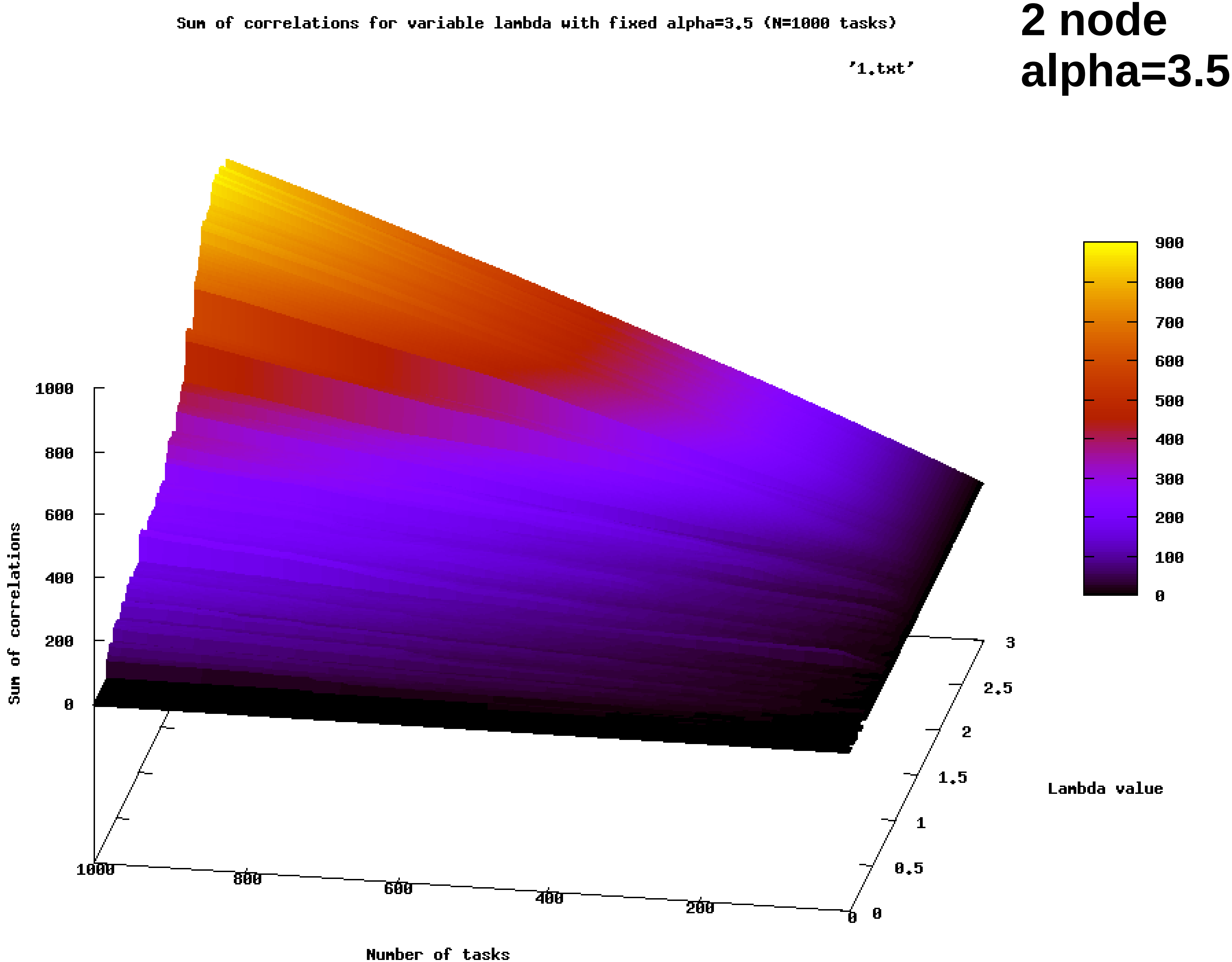
- C++ (STL)
- Intel Cluster Toolkit
- Boost (Boost::MPI)
- Gnuplot

- HPC @ IAMR KRC RAS (851 Gflops, 80 cores Xeon 2.66, 512mb/core, 1Tb SAN, OpenSUSE)
- PC (Intel Cel.2.66, 512mb, Zenwalk)

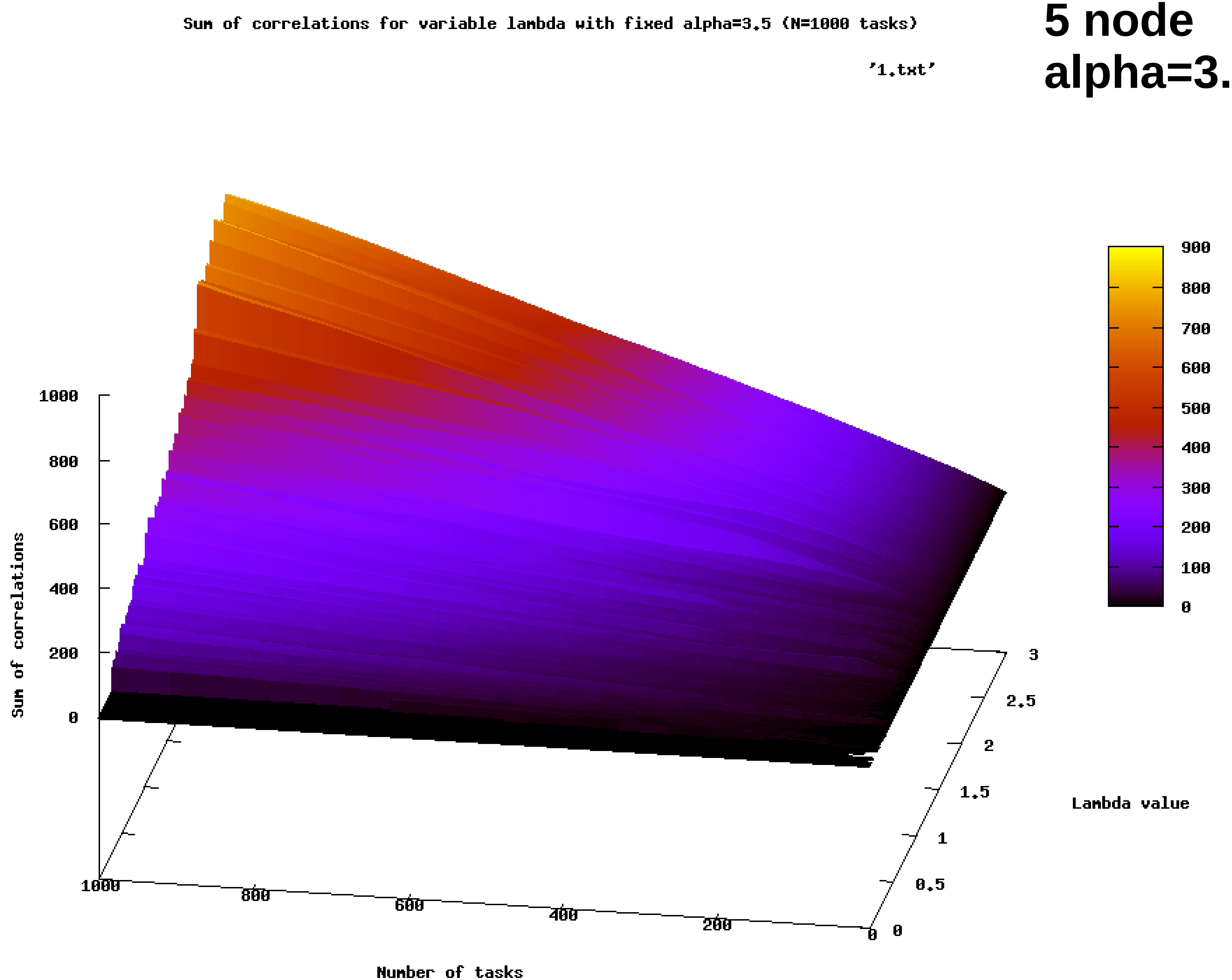
1 node alpha=3.5

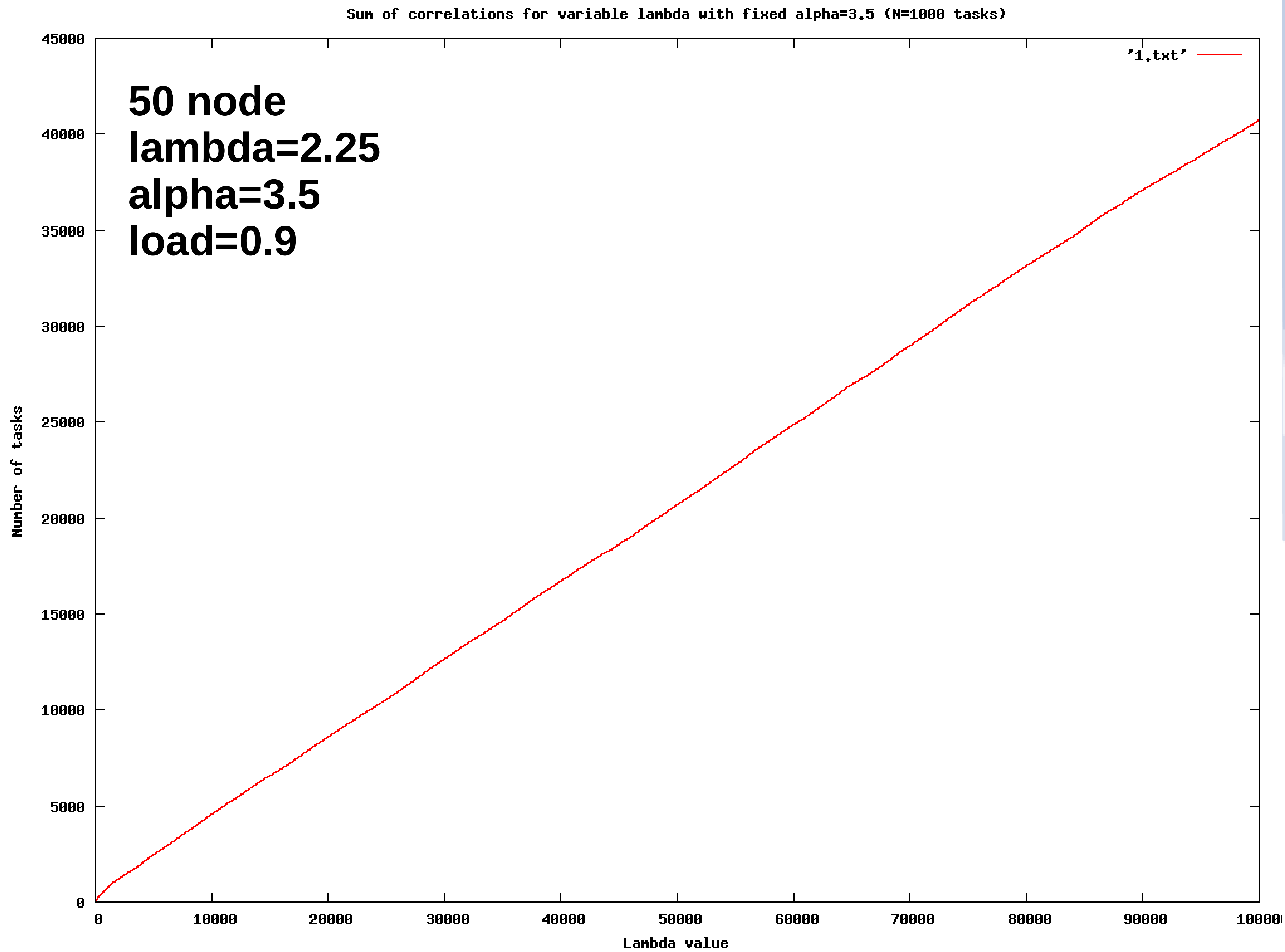


2 node alpha=3.5

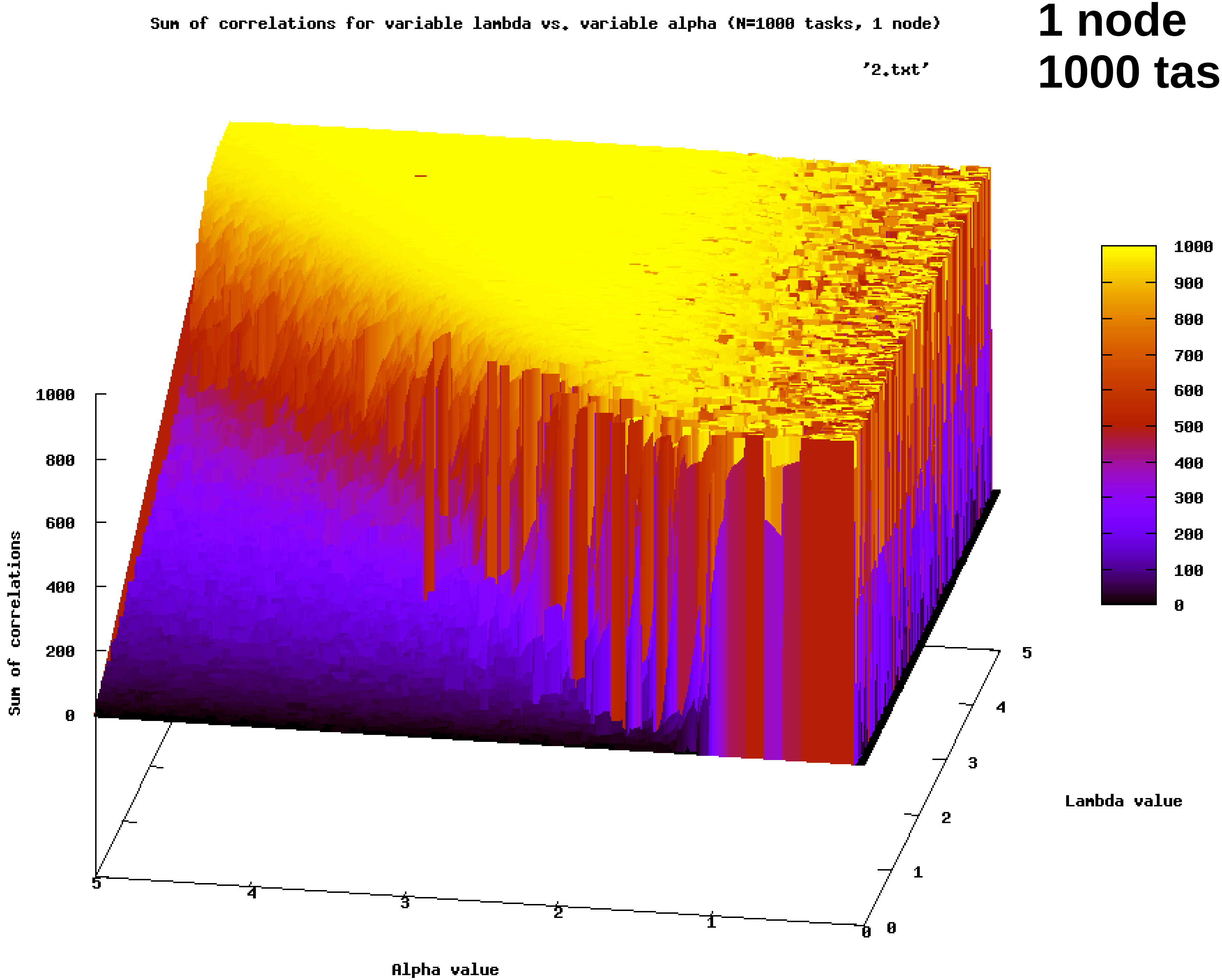


5 node alpha=3.5




Sum of correlations for variable lambda with fixed alpha=3.5 (N=1000 tasks)
'1.txt'
50 node
lambda=2.25
alpha=3.5
load=0.9
Number of tasks
Lambda value
0
5000
10000
15000
20000
25000
30000
35000
40000
45000
0
10000
20000
30000
40000
50000
60000
70000
80000
90000
100000

1 node
1000 tasks



Conclusion

- Empirical acknowledge of Daley results
- Extended interval of parameters, see the same behavior of correlation sums
- MPI routine for the sake of modeling purposes

Future research

- More complicated networks
- Networks with losses
- Large finite buffer, load >1
- Diverse parameters/distributions
- Regeneration cycles
- Other service disciplines

- Use MKL

Bibliography

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- T.Huang, K.Sigman. Steady-state asymptotics for tandem, split-match and other feedforward queues with heavy tailed service. 1999.
- A.Scheller-Wolf. Further delay moment results for FIFO multiserver queues. 1999.
- A.Scheller-Wolf, K.Sigman. Moments in tandem queues. 1996.