Gaussian processes in communication Networks

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Petrozavodsk,
May 19, 2009

Performance issues of communication Networks:

- Self-similarity of network traffic
- Broadband traffic flows exhibit Long Range Dependence
- Possibility of accurately modeling network traffic
- Justification of network traffic models

Properties that network traffic usually obeys:

- Stationarity

$$A(s,t) = A(0, t-s)$$

- High aggregation level

- Extreme irregularity of the traffic rate

$$R(t) = \lim_{s \uparrow t} \frac{A(s,t)}{t-s}$$

Gaussian sources:

$$A(s,t) =_d \mathbb{N}(\mu \cdot (t-s), \upsilon(t-s))$$

$$A(t) := A(0,t)$$

Covariation function:

$$\Gamma(s,t) := Cov(A(s), A(t)) = \frac{1}{2}(\upsilon(t) + \upsilon(s) - \upsilon(t-s))$$

Two fundamental properties of Gaussian sources:

$$C(t,\varepsilon) := Cov(A(0,\varepsilon), A(t,t+\varepsilon))$$

- Long-range dependence (lrd)

$$\sum_{k=1}^{\infty} C(k,1) = \infty$$

- Smoothness

$$\lim_{\varepsilon \downarrow 0} \frac{C(t,\varepsilon)}{\upsilon(\varepsilon)} \neq 0$$

Generic examples of Gaussian sources:

- Fractional Brownian motion (FBM)

$$\upsilon(t) = t^{2H}$$

  - Long-range dependent (LRD)
  - Nonsmooth

- Integrated Ornstein-Uhlenbeck (IOU)

$$\upsilon(t) = t - 1 + e^{-t}$$

  - Short-range dependent (SRD)
  - Smooth

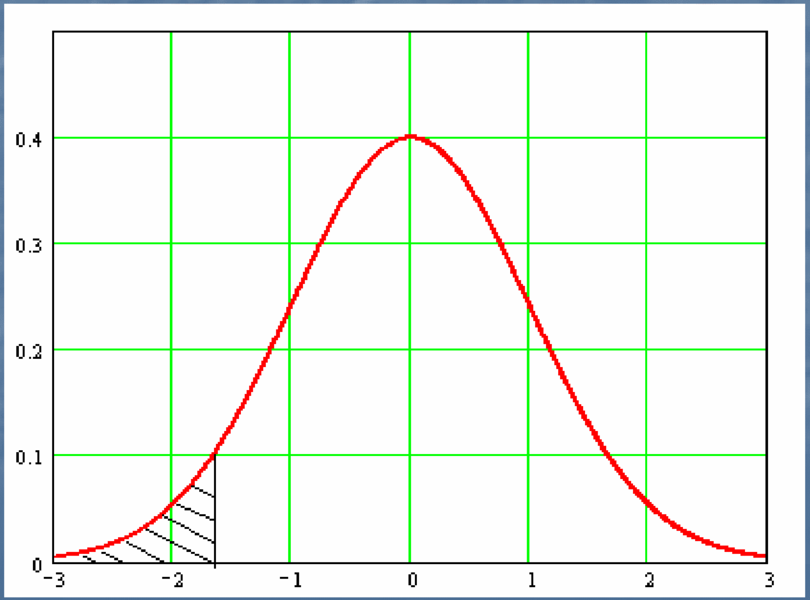
Applicability of Gaussian source models for network traffic :

- High aggregation level

$$A(t) \approx_d n\mu t + \sqrt{n\upsilon(t)}\,\mathbb{N}(0,1)$$

- Negative traffic

$$P(A(t) < 0) = P\left(\mathbb{N}(0,1) < -\frac{\sqrt{n\mu t}}{\sqrt{\upsilon(t)}}\right)$$



Simulation of FBM:

FBM as a sum of fractional Gaussian noises:

$$B_H(n) = \sum_{k=1}^{n} \beta(k)$$

Estimation of fractional Gaussian noise (FGN)

$$\widehat{\beta}_N(n) = \sum_{k=1}^{N} e^{i\lambda_k n} \cdot \Delta V_k$$

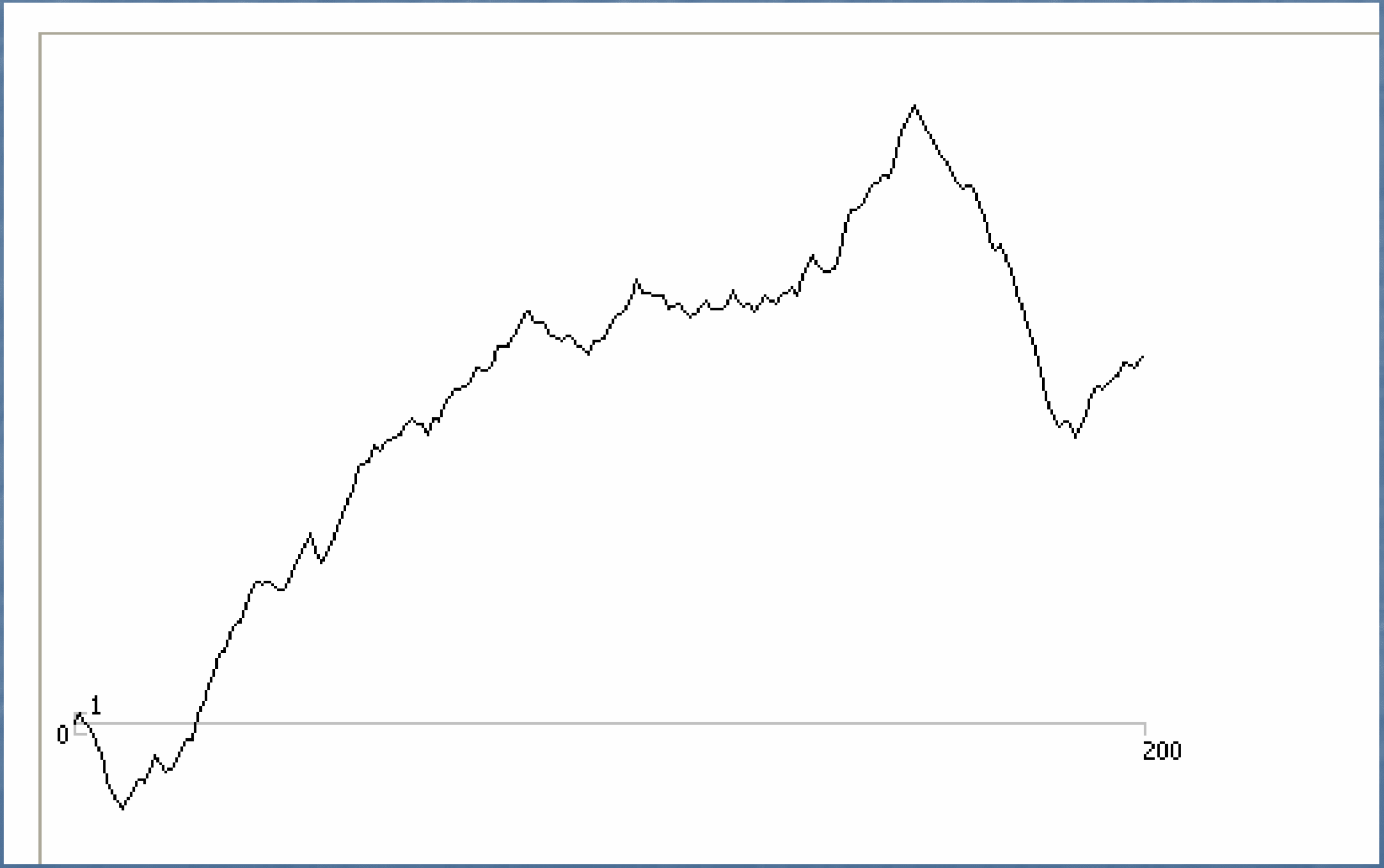
ΔV_k – complex-valued Gaussian random variable with variance:

$$\sigma_k^2 = \int_{\lambda_{k-1}}^{\lambda_k} f_H(w)\,dw$$

$f_H(w)$ – spectral density

Simulation example:

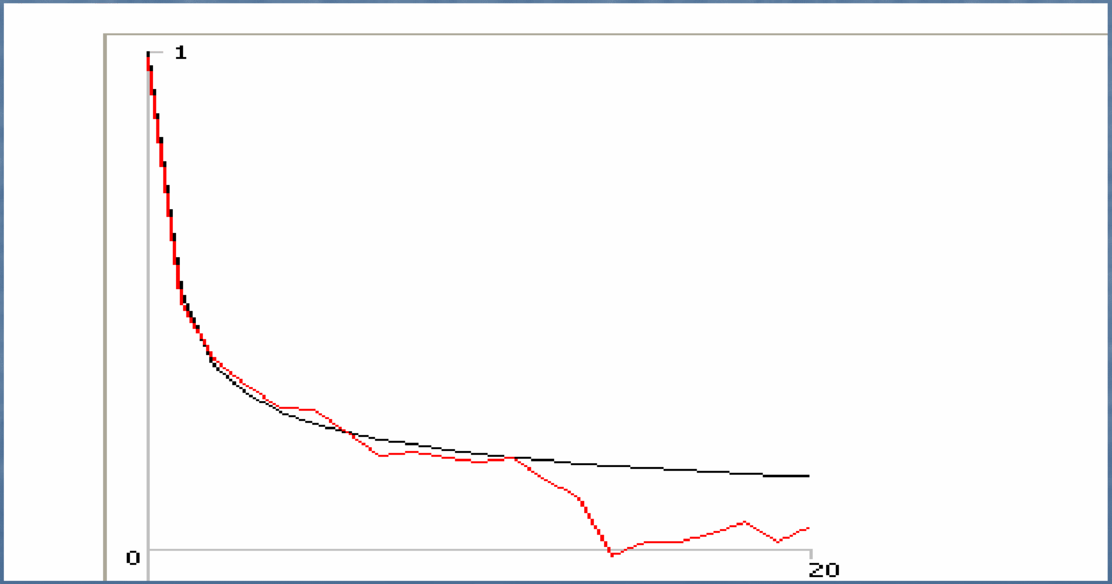
Realization of FBM (H=0,8):



Checking of simulation results:

Estimation of covariation function:

$$\hat{\rho}_S(n;x) = \frac{1}{S-n}\sum_{k=0}^{S-n-1} x_{n+k}x_k$$



Validation and justification:

- Central-limit type argumentation
  - Central limit theorem for on-off arrival processes
  - Special case of heavy-tailed on-off sources
- Statistical tests
  - N-Q plot
  - Goodness of fit test

Statistical test:

Null hypothesis:

$$H_0 : \; F = \Phi_{\mu,\sigma}$$

Difficulties:

- Observations can be dependent
- Unknown parameters of expected normal distribution

N-Q test:

Wasserstein metric:

$$\int_0^1 \left(F_n^{-1}(t) - \mu - \sigma\Phi^{-1}(t) \right)^2 dt$$

$$\hat{\mu} = \bar{x}$$

$$\hat{\sigma} = \int_0^1 F_n^{-1}(l)\Phi^{-1}(l)dl$$

Plotting positions:

$$a_i = \frac{\phi_{i-1} - \phi_i}{\sum_{i=1}^{n} (\phi_{i-1} - \phi_i)^2},$$

where

$$\phi_i = \phi\left(\Phi^{-1}\left(\frac{i}{n} \right) \right)$$

Goodness of fit test:

Correlation coefficient:

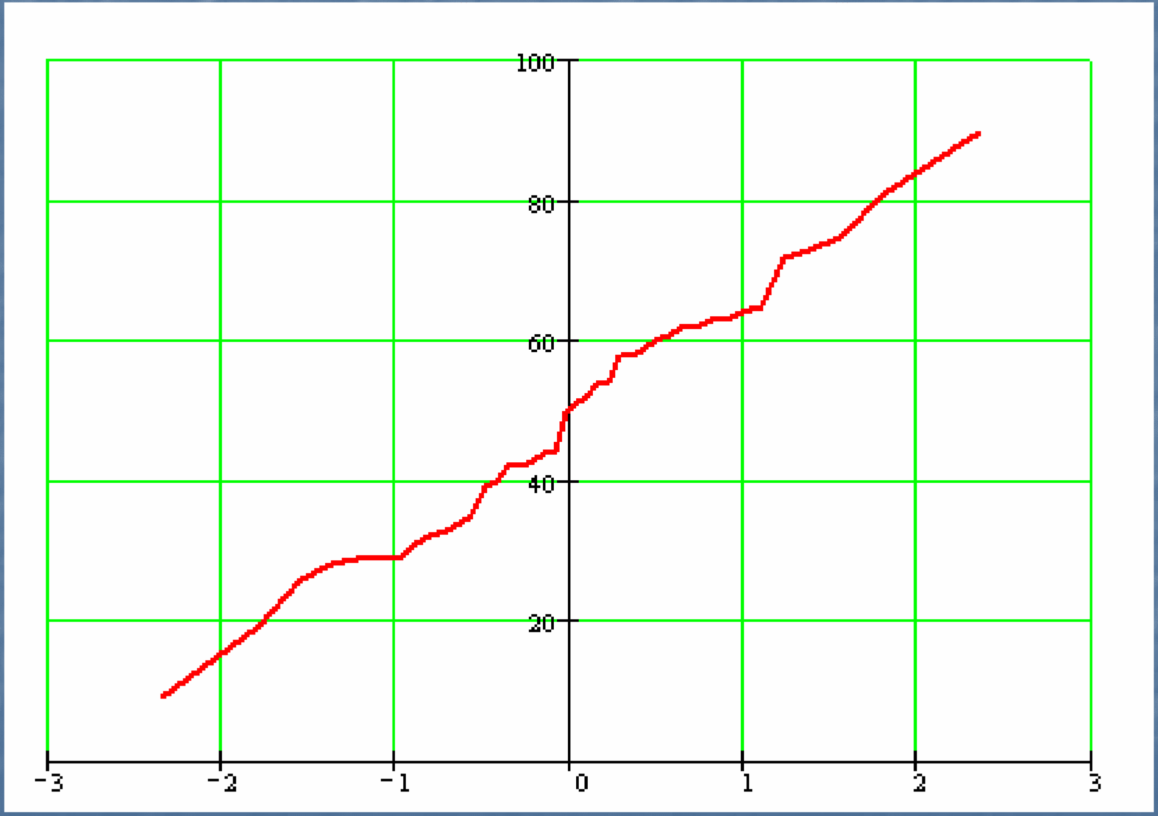
$$r = r(x,a) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(a_i - \bar{a})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (a_i - \bar{a})^2}}$$

Confidential interval:

$$\left[r - z_\gamma \frac{\sqrt{1-r^2}}{\sqrt{n-2}}, r + z_\gamma \frac{\sqrt{1-r^2}}{\sqrt{n-2}} \right]$$

Example and result from data:

N-Q plot:



Correlation coefficient:

$$r = 0,992$$

95% confidential interval:

$$[0,951;1,032]$$

Parameters:

$$\mu = 49,63 \quad \sigma = 17,44$$