Heavy tailed distributions with applications to broadband communication systems traffic

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What

- Our work is concerning the following:
  - ✳ Definitions
  - ✳ Properties
  - ✳ Applications
- All this connected with Heavy Tails, Self Similarity and Long Range Dependence

Where

- You can find Heavy Tails, Self Similarity and Long Range Dependence here:
  - LAN, WAN traffic measurements.
  - Single Processor: times of service.
  - Variable Bit Rate video traffic.
  - TCP traffic.
  - Insurance: claim size.
  - Number of signs on this slides.
- Lots of activities connected with human.

When

- **Heavy Tails** may occur when you see
  - ✳ Enormous variance.
  - ✳ Huge values of r.v. with big probability.
  - ✳ Infinite spectral density at the origin.
  - ✳ Linear curve on the log-log plot for the tail.
- **Long Range Dependence** may be when
  - ✳ Hurst phenomenon: slow decay of R/S statistics.
  - ✳ Autocorrelations decay slowly.
- **Self Similarity** waits for you when
  - ✳ On high levels of aggregation process doesn’t “smoothen”. It still has “pikes”.

Definitions

- Tail of distribution
- Heavy Tail (HT)
- Long Tail (LT)
- Subexponential Tail (ST)
- Regularly Varying Tail (RV(α))
- Long Range Dependence (LRD)
- Self-Similar Process (SS)

- $\overline{F}(x) = 1 - F(x)$
- $\forall \varepsilon > 0,\ E\{e^{\varepsilon X}\} = \infty$
- $\frac{\overline{F}(x+y)}{\overline{F}(x)} \to 1,\ x \to \infty$
- $\frac{P(X_1+X_2>x)}{P(X>x)} \to 2,\ x \to \infty$
- $\forall c > 0,\ \frac{L(cx)}{L(x)} \to 1,\ x \to \infty$
- $\overline{F}(x) = x^{-\alpha} L(x),\ \alpha > 0$
- ? phase transition
- $\forall c > 0,\ (Y(ct),\ t \geq 0) \sim c^H (Y(t),\ t \geq 0)$

Properties: RV

- The most convenient type of Heavy Tailed random variable.
- RV(α): the less the index α is, the "heavier" the tail is, the bigger is the probability for the random variable to have huge value.
- If index $\alpha<2$, then the dispersion is infinite, if index $\alpha<1$, then mean is infinite.
- The distribution of unfinished work (or equilibrium distribution, residual lifetime distribution) has tail 1— α heavier than the distribution of whole work.

Properties: SS

- Self Similarity on the application level is not important to be noticed while Self Similarity on the network layer is to be of great importance.
- The most interesting result (I. Norros): storage model: fractional Brownian input with mean input rate m, Self-Similarity parameter H and output at constant rate C. Let the probability of high buffer level be P(V>x)=ε. What is the behavior of level x while increasing the utilization level m/C?
  - ✴ Classical case H=1/2 (Brownian motion, no dependence).

$$\frac{1-\rho}{\rho}\cdot x = const.$$

    - Reducing by half free capacity 1-ρ costs doubling needed level x.
  - ✴ Self-Similar case 1/2<H<1

$$x = x(\rho) = const\cdot\rho^{1/(2(1-H))}\cdot(1-\rho)^{-H/(1-H)}.$$

    - Ex.: H=0.9. Reducing by half free capacity 1-ρ costs 512 times increase of x.

Relations: examples

- LRD may follow from HT. For instance, On-Off process is LRD if and only if On and Off times have infinite variance.
- Superposition of On-Off processes with Pareto distributed On and Off times with infinite variance produce Self-Similar traffic.

Why

- We need to study HT, LRD and SS because:
  - ✳ Definitions differ. Misunderstanding between authors may occur.
  - ✳ Consequences of wrong analysis may be rather dramatic for the finances of the company.
- So let us explore some applications.

Consequences of wrong analysis: example

- A single server is connected to k=99 client machines and works at a constant rate r=100. What will happen if the service rate increases as r′=1.1 r?
- M/M/1 model. Let the intensity of the client be equal to λ=1. Then the intensity of the input as a superposition of Poisson processes has an intensity 99. Then the load is equal to 99/100 and the stationary queue size is 99. For the service rate r′=110 the load is 99/110 and the queue size is 9 that is **11 times less**! So the upgrade is essentially needed!
- On-Off model. The source alternates between two states: transmitting and waiting. The time for On and Off states have RV distributions, Off has a lighter tail. Let the average time spent in On state be 1 and in Off state be 100 (i.e. the client silently edits the document for a long time and then sends it for a short time). Then the intensity of transmission is a=101 from the equation $\lambda = \frac{\mu_{on}}{\mu_{on} + \mu_{off}} a$. The crucial parameter is $\kappa_0 = \left\lceil \frac{r}{a} \frac{\mu_{on} + \mu_{off}}{\mu_{off}} - k \frac{\mu_{on}}{\mu_{off}} \right\rceil$ related to the time to overflow in system with finite buffer. If it changes, then the performance also changes. But the parameter is equal to 1 in case r=100 and r′=110. So the upgrade **doesn't change anything**. Moreover, one can count the r″ needed to change the parameter, and can find r″>199 that is **increase of service rate in about two times!**

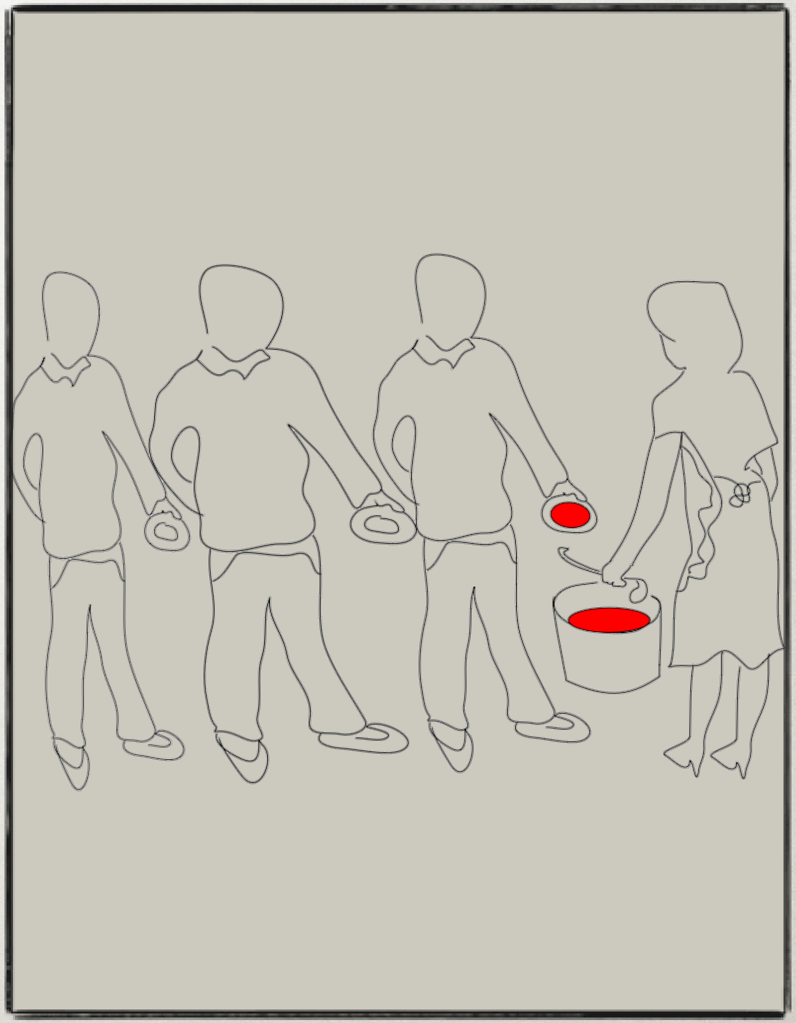
What's the difference?

- M/G/1 model: compare three service disciplines: First Come First Serve, Last Come First Serve, Processor Sharing. The event of large sojourn time is caused in different ways.
- k different sources. Let the service time distributions be Regularly Varying with indexes $\nu_1 < \nu_2 < \ldots < \nu_k$

First Come First Served (FIFO)

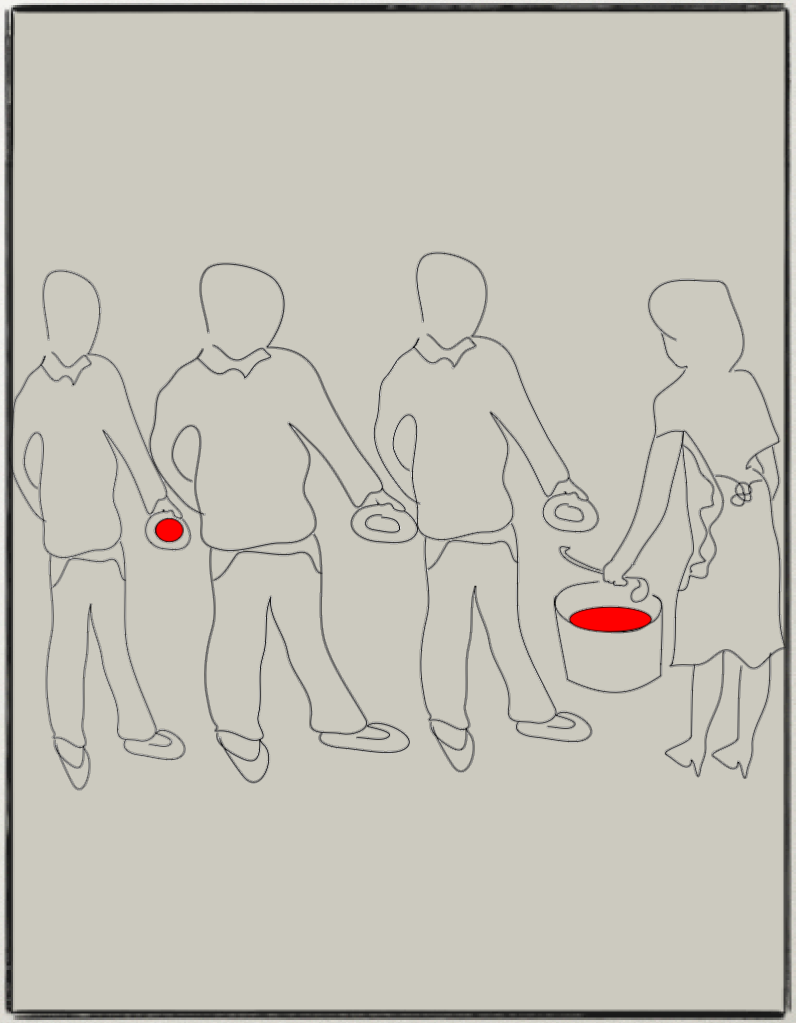
- RV(ν_i) service causes sojourn time RV($1 - \nu_1$) even heavier than the heaviest service tail.

- Long sojourn time due to **unfinished service** of some task with the heaviest tail that comes before.



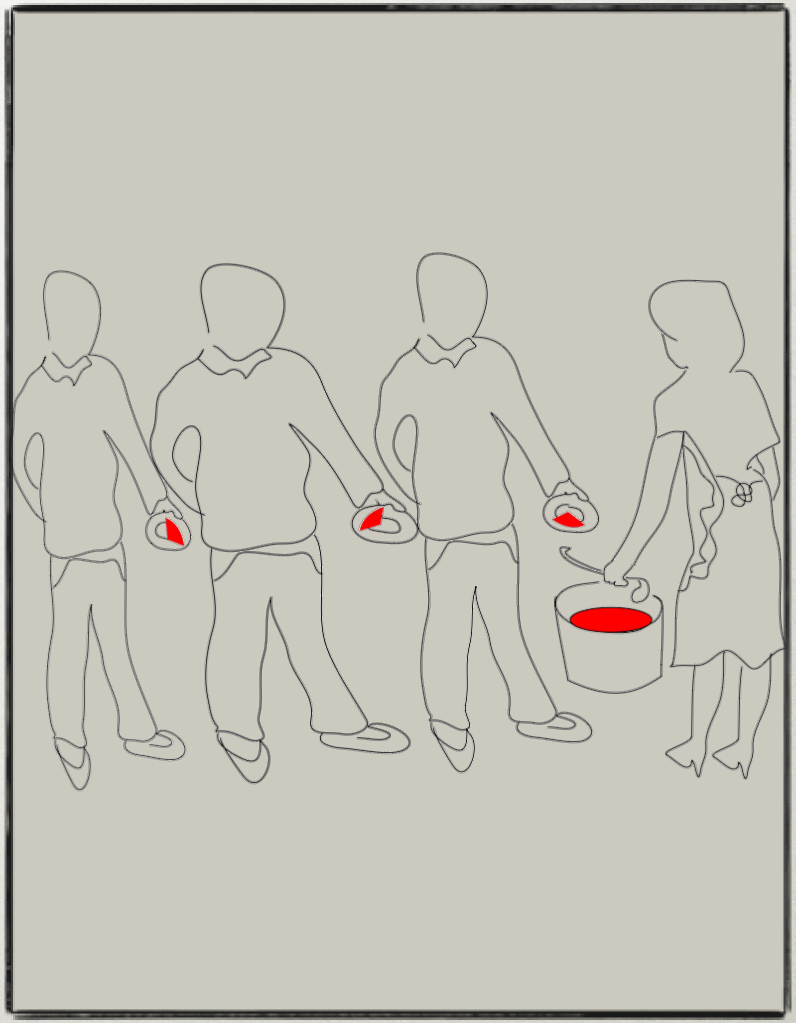
Last Come First Served (LIFO)

- RV(ν_i) service causes sojourn time RV(ν_1) equally heavy to the heaviest service tail.
- Long sojourn time because of later **service** of some task with the heaviest tail, that comes **after** this.



Processor Sharing

- RV(ν_i) service causes equally heavy RV(ν_i) sojourn time.
- Long sojourn time because of it's **own long service** time. Huge tasks don't affect the performance too much.



Upgrade?

- 10 Mbit Ethernet => 1.3 Mbps average service level
- 100 Mbit ATM net => 0.3 Mbps
- Fast network: TCP manages to send more bits before changing window size, so the collision becomes only worse.
- So why upgrade?
- Networks are to be deeply investigated.

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