Models of Inventory Management of Industrial Enterprise in view of Interchangeability of Materials and Variability in Delivery Sizes due to Vehicle Capacities

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Abstract

We present two models using mixed linear programming. The first considers a set of materials which may be interchanged and obtained from several suppliers. We study the planning of deliveries with a number of interrelated goals, such as:

1) defining the most economical plan of supplying the demand, taking account of the interchangeability of materials and of the unused stock of materials in warehouses;

2) determining the order vector for the delivery, dimension of which can be reduced due to replacement of the materials and which takes into account need on restoration of a stock to the beginning of the next period;

3) determining the types of vehicle necessary to deliver the orders, the sizes of the vehicles and its number.

The total expense of the enterprise, including the costs of replacement of materials, the organization of deliveries, and carriage and the additional cost if the material is not purchased from the producer, is minimized in the objective function. In the second model, the content of deliveries and the timing of delivery of the materials are modeled.

Material maintenance of production in view of Interchangeability of Materials and Variability in Delivery Sizes due to Vehicle Capacities

Introduction

Delivery planning essentially affects the expenses of an enterprise, which are connected with production material maintenance (MM).

Within multinomenclature production, many materials are needed in only small amounts of kilos and dozens of kilos. The delivery of all the materials listed according to design and technology requirements, including thousands of brands, types, sorts , and sizes, is quite laborious and not always necessary and profitable. For example, at the examined enterprise, there are about 1600 different metal-rolls listed in the technology documents; actually, less than 400 units are delivered to the plant. The rest of them, mostly those that are needed in small amounts, are usually substituted by the metals needed in larger amount, which are of the same quality and sometimes of the same brand, but with large allowance. If there are any doubts, such changes are submitted to the design engineers' approval.

Supply engineers try to reduce the number of the ordered materials (to unify the order) by means of substituting several similar material units by one of them. As well as in order to reduce the expenses for unused remains maintenance, the delivery costs, and the specific transportation expenses by means of increasing delivery.

On the contrary, production requires all the materials listed according to the technology documents. Denying of these requirements could lead to additional expenses, because, due to extra allowances and losses during the laborious processing of the substitutes, the amount of in-process waste would increase. Since quality of the substitute material must be as high as that of the substituted material, its cost sometimes is higher, too.

Therefore, material consumption and its processing laboriousness and cost may increase. We will call these additional expenses 'substitute costs'.

The spendings of an enterprise are substantially affected by transport storing expenses (TSE). For small-tonnage deliveries, specific TSE are usually much higher than those for large-tonnage deliveries.

The specific TSE are lower for railroad deliveries received directly from the producers, than for truck haulage. In addition, producers sell their materials at wholesale prices. On the contrary, supply storehouses add their extra charges to the initial prices. However, there are some organization limits for railroad deliveries. A freight car can be loaded either entirely or according to the transit standards. For metal-roll, it is 65 and 20 tons. Besides, there are some minimum requirements of limits of orders, when the same metal-roll can't be loaded into a railroad car in the quantity less than 5 tons. Truck haulage has no such limits.

On the other hand, if materials of the same type are delivered to an enterprise in large amounts exceeding the actual needs, the storage costs increase – because of both warehousing and floating assets freezing.

If the production process is carried out without sufficient supply resources, deficiency may take place, and expensive urgent delivery may be needed.

So, different factors affect the costs of production MM in opposite ways. That is why, when we are planning MM and material delivery, it is necessary to seek solutions that would minimize the total expenses of:

- material substitution;
- delivery organization and material shipment to the enterprise;
- storing materials at the enterprise;
- warehouse extra charges;
- urgent delivery in case of deficiency;
- unused materials storage and other factors.

These expenses can be minimized only by means of optimization mathematical model. Below we suggest a variant of such model.

Statement of the problem

For example, an enterprise tries to have certain supply resources by the beginning of each interval of planning period (a month in a quarter, a week in a month, a day in a week). So resources amount is equal to the demand of interval of the period or is proportional to the needs of the period it is made for. Let us consider that the needs for the planned and post-planned periods according to the design and technology requirements are calculated and known to us. It's necessary to find:

- the most efficient ways of meeting the needs of the period considering interchangeability of materials;
- the vector of the ordered materials considering the expected remains at the enterprise's stores and requirements of restoration of a stock to the beginning of the next period;
- the distribution of the order vector upon the transit and store components considering the common suppliers;
- the number of deliveries from each supplier considering the capacities of the used vehicles.

Let us introduce there following symbols:

M the multitude of the interchangeable materials that are bought from the common suppliers, i,j$\in M$;

M_j - the multitude of the materials that can substitute j-material, $M_j \subset M$;

N_i - the multitude of the materials that can be substituted by i-material, $N_i \subset M$;

S - the multitude of the suppliers that provide the materials of M group, $s \in S$;

M^s - the multitude of the materials that can be bought from s - supplier;

S^r - the multitude of the transit suppliers of materials;

S^w - the multitude of the storehouse suppliers of materials, $S^r \cup S^w = S$;

S_i - the multitude of the suppliers of i-material;

K - the multitude of delivery amounts corresponding to the capacities of the vehicles, $k \in K$;

K^r - the multitude of transit delivery amounts, $K^r \subset K$;

K^w - the multitude of storehouse delivery amounts, $K^w \subset K$, $K^r \cup K^w = K$;

K_s - the multitude of delivery amounts made by s - supplier;

U_i - the expected remains of i-material at the enterprise by the beginning of the period;

D_i^1 - the enterprise's needs in i - material for the planning period;

D_i^2 - the enterprise's needs in i - material for the post-planning period;

L_i - the minimal ordered amount of i-material by railroad;

P - some quantity big enough not to be exceeded by the demand for any material;

P_s - the total need of the period for the materials provided by s - supplier;

C_{ks} - the delivery amount or capacity of k - type from s - supplier;

C_i - the wholesale price of i -material;

F - the amount of floating assets assigned for buying materials;

R_{ij} - the expenses of substituting j-material by i-material accounting for one substitute unit;

E_s - the specific expenses of storing the materials provided by s - supplier during the period;

E_i^u - the specific expenses of storing the materials exceeding the needs in i-material;

E_{ks} - the total expenses of organization, delivery and storage of the supplies of k - rate from s - supplier; the calculation formula is given below;

T_i - the specific TSE of restoring the normal required minimum stock of i-material;

T_{ks} - transportation expenses and delivery organization expenses of k - type supply from s - supplier;

T_i^e - the specific expenses of urgent delivery for deficiency elimination;

B_i - the fixed expenses in case of deficiency of i-material;

α_i - the share of the normal required minimum stock by the beginning of the period from the total need of period in i - material;

β_{ij} - the factor of substituting j - material by i - material, it shows which part of the unit in j - material is compensated by the one unit of i-material; as a rule, it is less than one; $\beta_{jj} = 1$ for all j;

γ_i - the share of storehouse extra charge in the wholesale price of i-material;

θ - the average share of TSE in the price of the shipped materials.

It is necessary to find:

r_{ij} - the amount of i-material assigned for substituting j - material;

r_i - the need in i - material for the period considering the planned substitution;

o_i - the total amount of the order of i-material;

o_i^r - the total amount of the order of i-material by railroad;

o_i^w - the total amount of the order of i-material by the storehouse type of delivery;

o_{is} - the amount of i-material ordered from s - supplier;

q_{ks} - the number of deliveries of k - rate from s - supplier;

s_i - the stock of i-material by the beginning of the post-planned period;

s_i^l - the quantity of deficiency in the normal required stock in i-material by the beginning of the post-planned period;

s_i^u - the stock exceeding the normal need in i-material by the beginning of the post-planned period;

s_i^o - the deficiency in i-material for the planning period;

δ_i - Boolean variable, in the unity status provides the order of i-material by transit in the amount not less than the ordered norm; otherwise the refusal from ordering this material;

e_i - Boolean variable, the unity status of which means that there is deficiency in i-material.

Model Development

Possible variants of meeting the needs of production for the planning period considering the materials interchangeability we will write as follows:

$$\sum_{i \in M_j} \beta_{ij} r_{ij} >= D_j^1 \qquad \forall \quad j \in M \tag{1}$$

The total need in i- material for production maintenance we will write as follows:

$$\sum_{j \in N_i} r_{ij} - r_i = 0 \qquad \forall \quad i \in M \tag{2}$$

For the materials that are provided by several transit suppliers, we will find the total amount of i - material delivery from the following equations:

$$\sum_{s \in S^r} o_{is} - o_i^r = 0 \qquad \forall \quad i \in M \tag{3}$$

The amounts of the stocks of i- material by the beginning of the post-planned period, the amount of the order by both transit and storehouse types of delivery, as well as the quantities of the expected deficiency we will find from the following in equations:

$$r_i + s_i - o_i^r - o_i^w - s_i^o \le U_i \qquad \forall \quad i \in M \tag{4}$$

The amount of the stock exceeding the normal required minimum of i - material we will find from the following limitations:

$$r_i + s_i - o_i^r - o_i^w + s_i^u \le U_i \qquad \forall \quad i \in M \tag{5}$$

The amount of the stock and the amount of deficiency in it we will find from the following equations:

$$s_i + s_i^l = \alpha_i D_i^2 \qquad \forall \quad i \in M \tag{6}$$

The following limits allow us to remember in which types of materials there is deficiency:

$$s_i^o - Pe_i \le 0 \qquad \forall \quad i \in M \qquad (7)$$

The amount of materials delivered by transit suppliers and the number of deliveries from its, we will find from the following equations:

$$\sum_{i \in M^s} o_{is} - \sum_{k \in K^r} C_{ks} q_{ks} = 0 \qquad \forall \quad s \in S^r \qquad (8)$$

The demand of meeting the normal minimal order for transit suppliers we will write as follows:

$$L_i \delta_i \le o_{is} \le P \delta_i \qquad \forall \quad i \in M \qquad (9)$$

The amount and number of storehouse deliveries we will find from the following in equations:

$$\sum_{i \in M^s} o_{is} - \sum_{k \in K^r} C_{ks} q_{ks} \le 0 \qquad \forall \quad s \in S^w \qquad (10)$$

If the amount of the floating assets of the enterprise is limited, the following limitation should be included in the model:

$$\sum_{i \in M} (1+\theta)(C_i o_i^r + (1+\gamma) C_i o_i^w) \le F \qquad (11)$$

In the objective function of the task, the expenses of the following are minimized: material substitution, transportation and organization of delivery and storage of current stock, storehouse extra charges, urgent delivery in case of deficiency in stocks, storage of exceeding stocks, urgent delivery in case of deficiency in meeting needs, the fixed expenses of deficiency. Therefore, we will write the objective function as follows:

$$\min\{ \sum_{j \in M} \sum_{i \in M_j} R_{ij} r_{ij} + \sum_{s \in S} \sum_{k \in K} E_{ks} q_{ks} + \sum_{i \in M} \gamma_i C_i o_i^w + \sum_{i \in M} (T_i s_i^l + E_i^u s_i^u + T_i^e s_i^o) \} + \sum_{i \in M} B_i e_i , \qquad (12)$$

where E_{ks} - we will calculate according to the formula: $E_{ks} = T_{ks} + \frac{1}{2} \frac{E_s C_{ks}^2}{P_s}$, (13)

supposing that $P_s \approx \sum_{k \in K_s} C_{ks} q_{ks}$. The second item in (13) denotes the expenses of storing the average stock $\frac{1}{2} C_{ks}$ in the time of $\frac{C_{ks}}{\sum_{k \in K_s} C_{ks} q_{ks}}$ under perfect management,

that is the next delivery is made when the last one is exhausted with uniform consumption.

Calculation Sample

Using the model, we have made optimization calculations on PC by means of lp_solve. The input data are given in table 1. The amount of material is given in tons, the price in conventional units (roubles at the moment of research) per ton. The normal minimum level of stock is taken as equal to 0.333 of the need of the second period. The results of optimization are in tables 2 and 3.

Table 1

Mate-rial №	Band №	Diameter, Thickness	Wholesale Price	Remains at the Enterprise	Demand for the 1st Period	Demand for the 2nd Period	Transit Suppliers №	Warehouse Supplier №
1	1	8	661	0	0.695	5.878	2	1
2	1	10	661	0	3.957	15.832	2	1
3	1	12	863	0	0.050	0.938	3	1
4	1	16	864	20	87.141	134.621	3	1
5	1	18	864	40	92.104	105.460	3	1
6	1	20	864	70	127.913	194.678	3	1
7	1	22	864	60	163.930	163.930	4; 5	1
8	1	26	868	20	71.594	71.594	4	1
9	2	10	1100	0	4.964	11.560	3	1
10	2	16	939	0	1.251	39.841	3	1
11	2	18	939	0	3.780	12.671	3	1
12	2	20	939	0	7.560	65.195	3	1
13	3	10	1130	0	3.580	0	3	1
14	3	12	1130	0	8.712	37.925	3	1
15	4	10	1240	0	5.087	0	3	1
16	4	12	1240	0	0.090	0.933	3	1
17	4	36	1250	120	321.832	214.235	4; 5	1
Total				330	904.240	1075.291		

Table 2

Supplier №	Supply amount	Number of deliveries	Delivery amount	Delivery №
1	3	1	3	1
1	5	0	0	0
1	8	0	0	0
1	10	0	0	0
1	14	0	0	0
2	20	1	20	2
2	65	0	0	0
3	20	1	20	3
3	65	6	390	4-9
4	20	0	0	0
4	65	2	130	10-11
5	20	0	0	0
5	65	6	390	12-17
Total		17	953	17

Table 3

Material №	Demand for the 1st Period considering the Stock and deducting the Remains	The Planned Amount of Order	Including the Substitute / Substitute №	Including the Stock for the Second Period	Suppliers №№	Train Delivery	Car Delivery
1	2	3	4	5	6	7	8
1	2.654	5.000		1.959	2	5.000	0.000
2	9.234	15.000		5.277	2	15.000	0.000
3	0.363	0.363		0.313	1	0.000	0.363
4	112.015	112.017		44.874	3; 1	109.470	2.547
5	87.257	87.257		35.153	3	87.257	0.000
6	122.806	122.810		64.893	3	122.810	0.000
7	158.573	171.301		54.643	4	54.541	0.000
7					5	116.760	
8	75.459	75.459		23.865	4	75.459	0.000
9	8.817	7.397	1.420/13	3.853	3	7.397	0.000
10	14.531	14.336		13.280	3	14.336	0.000
11	8.004	8.004		4.224	3	8.004	0.000
12	29.292	29.292		21.732	3	29.292	0.000
13	3.580	5.000		0.000	3	5.000	0.000
14	21.354	21.354		12.642	3	21.354	0.000
15	5.087	5.087		0.000	3	5.087	0.000
16	0.401	0.090		0.000	3	0.000	0.090
17	273.244	273.240		71.412	5	273.240	0.000
Total	932.671	953.007	1.420	357.036		950.007	3.000

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