PRESENTATION

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An alternative approach to TCP performance analysis

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Abstract

In this paper we present original approach to the TCP performance evaluation. With the aim we have formulated semi-Markovian process that describes behavior of TCP New Reno implementation. We have explicitly solved corresponding Kolmogorov equations and obtained the analytical presentation for congestion window size [1]. Since the solution close form is cumbersome we compute its approximation of the second order. The approximation is built as for the probabilities that form congestion window size distribution, so for its expectation and variance. The approximation allows analyst to use simple algebraical presentation of these values. The evaluated formulae doesn't tend to infinity as segment loss probability tends to zero and counts window size limitations and RTT variability.

Motivation and related work

A significant amount of today's Internet traffic including WWW (HTTP), file transfer (FTP), email (SMTP), and remote access (Telnet) traffic, is carried by TCP transport protocol. TCP together with UDP form the very core of today's Internet transport layer. Several efforts have been directed at analytically characterizing the throughput of TCP's congestion control mechanism. One reason for this recent interest is that simple quantitative characterization of TCP throughput offers the possibility of defining a "fair share" or "TCP-friendly" throughput for a non-TCP flow that interacts with a TCP connection. Another reasons are the necessity of performance evaluation, capacity planning and enhancement development of transport layer protocol.

All these reasons motivate significant efforts that have been directed towards developing the models for analytical evaluation of TCP throughput [2] — [4]. Most of these researches provide an estimation of average TCP throughput.

Disadvantages of previous approaches

One of the most referenced TCP throughput estimations is well-known Floyd formulae [3]:

$$B(p) = \frac{1}{RTT}\sqrt{\frac{3}{2bp}} + o(1/\sqrt{p})$$

However, it has some significant disadvantages:

- congestion window size tends to infinity as loss probability tends to zero;
- RTT is considered as a deterministic variable;
- window size limitations aren't counted.

Formulation of the model

Let $w(t)$ be the size of the sliding window w at instant $t > 0$ and let τ_i be a sequence of the instants at which the window w changes due to the operation of the AIMD algorithm. Then the sequence $w_i = w(\tau_i), i = 1, 2, \dots$, takes values from the set $X = \{2, 3, \dots, w_{max}\}$ and, by our assumptions, is a Markov chain. The minimal window size defined by the AIMD algorithm is 2. The step-wise random process $\{w(t)\}_{t>0}$ is semi-Markovian. Let p^k_{uv} be the transition probability for the chain w_i to pass from the state u to the state v in k steps, $u, v \in X$. According to our assumptions, $p^k_{uv} \xrightarrow[k\to\infty]{} \pi_v$. Let us denote $P_w(t) = \{w(t) = w\}$ and $\alpha_w = E[\tau_{i+1} - \tau_i | w(\tau_i) = w]$. Figure 1 illustrates our approach and its distinction with approaches based on the 'triple-duplicate' periods producing so called 'root-square' lows. On the figure the periods beginnings are marked td_i.

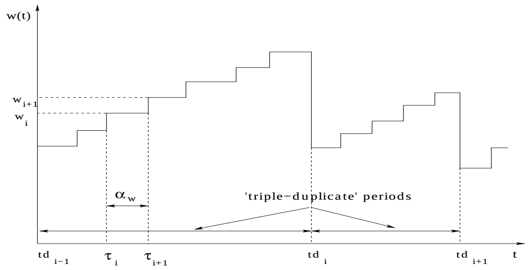

Figure 1: The step-wise random process of the congestion window size.

Second order approximation of the main performance characteristics

Let us consider semi-Markovian process that describes behavior of TCP Reno implementation and corresponding Kolmogorov equations:

$$\begin{cases} p_k = q^{k-1}p_{k-1} + (1-q^{2k})p_{2k} + (1-q^{2k+1})p_{2k+1}, & 2 < k < a \\ p_k = q^{k-1}p_{k-1} + (1-q^{2k})p_{2k}, & k = a \\ p_k = q^{k-1}p_{k-1}, & a < k < n, \\ p_k = q^{k-1}p_{k-1} + q^k p_k, & k = n, \\ \sum_{i=2}^{n} p_i = 1. & \end{cases}$$

The system has the following solutions:

$$\begin{array}{ll} p_k = \left(\dfrac{1-F_{2a-1}}{\prod_{i=k}^{a-1} q^i} + \displaystyle\sum_{i=k+1}^{a-1} \dfrac{F_{2i+1} - F_{2i-1}}{\prod_{j=k}^{i-1} q^j}\right) p_a, & 2 < k < a, \\ p_k = F_{k-1}p_a, & a < k < n, \\ p_k = \dfrac{F_{k-1}}{1-q^k}p_a, & k = n. \end{array}$$

where $F_i = \prod_{k=a}^{i} q^i$.

For illustration, let's choose $n = 10$. We derive the following second order approximations of solutions:

$$\begin{aligned} p_2 &= 50p^2 + o(p^2); \\ p_3 &= 180p^2 + o(p^2); \\ p_4 &= 350p^2 + o(p^2); \\ p_5 &= 10p - 145p^2 + o(p^2); \\ p_6 &= 10p - 245p^2 + o(p^2); \\ p_7 &= 10p - 305p^2 + o(p^2); \\ p_8 &= 10p - 375p^2 + o(p^2); \\ p_9 &= 10p - 455p^2 + o(p^2); \\ p_{10} &= 1 - 50p + 995p^2 + o(q^2), \end{aligned}$$

and the following approximation of TCP throughput:

$$E(p) = \sum_{i=2}^{10} i p_i = 10 - 150p + 235p^2 + o(p^2).$$

Figure 2 illustrates empirical TCP throughput, second order approximation of TCP throughput and TCP throughput estimated by Floyd [3] formulae.

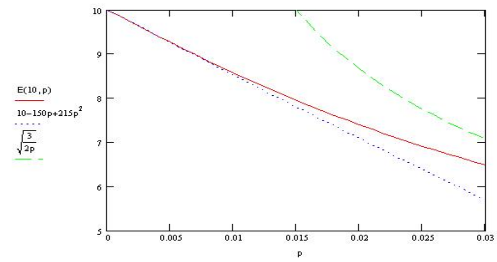

Figure 2: Different approximations of the congestion window size.

References

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