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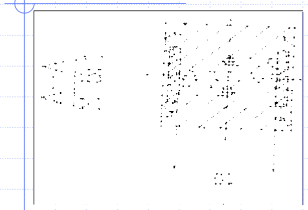
Efficient algorithms for polygonal approximation

Alexander Kolesnikov

Examples of polygonal approximation



Vectorization

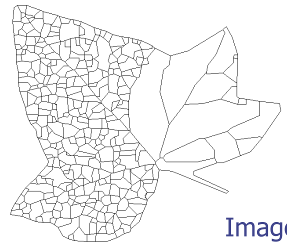
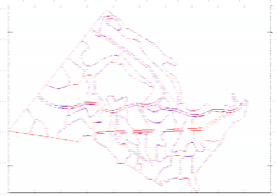
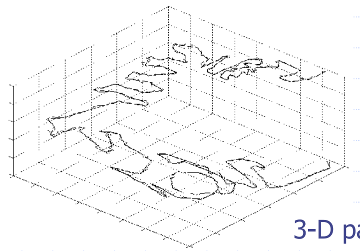


Image analysis



Digital cartography



3-D paths

Heuristic algorithms for approximation

1) Sequential tracing approach
2) Split method
3) Merge method
4) Split-and-Merge method
5) Dominant point detection
6) Relaxation labeling
7) *K*-means method
8) Genetic (evolutional) algorithms
9) Ant colony optimization method
10) Tabu search
11) Discrete particle swarm algorithm
12) Vertex adjustment method

Min-ε problem: Motivation

Heuristic algorithms
Non-optimal: F<100%
Fast: $O(N)$–$O(N^2)$
(seconds and less)

Optimal algorithm
Optimal: F=100%
Slow: $O(N^2)$–$O(N^3)$
(minutes and more)

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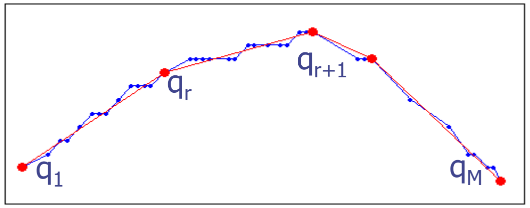
Fast: O(N)–O(N^2)
(seconds and less)

Optimal algorithm
Optimal: F=100%
Slow: O(N^2)–O(N^3)
(minutes and more)

Efficent algorithm
Fast: O(N)–O(N^2)
Close to optimal: F≅100%

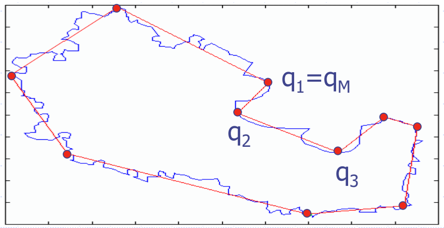
Min-ε problem for open curve



Approximate the given open N-vertex polygonal curve P by another one Q consisting of at most M line segments with minimum error E(P):

$$E(P) = \min_{\{q_r\}} \sum_{r=2}^{M-2} e^2(q_r, q_{r+1})$$

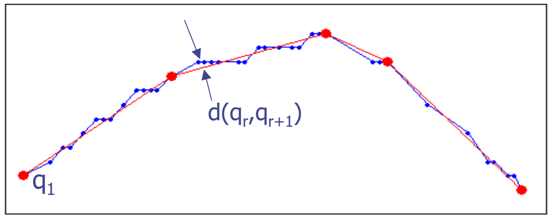
Min-ε problem for closed curves



Approximate the given closed N-vertex polygonal curve P by another one Q consisting of at most M line segments with minimum error E(P):

$$E(P) = \min_{\{q_r\}} \sum_{r=1}^{M-1} e^2(q_r, q_{r+1})$$

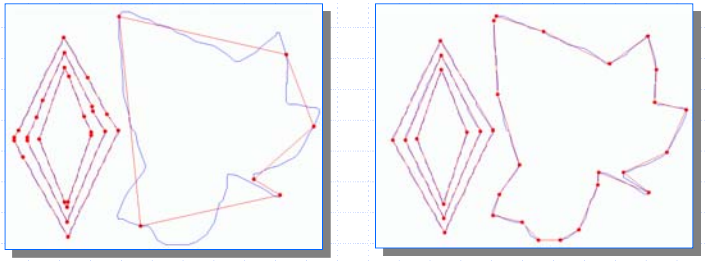
Min-# problem for open curve



Given polygonal curve P, approximate it by another polygonal curve *Q* with the minimum number of segments M so that the approximation error (distortion) does not exceed a given maximum tolerance ε:

$$M \rightarrow \min \quad \text{subject to: } d(P) \leq \varepsilon$$

Multi-object *min-ε problem*



Given K polygonal curves $P_1, P_2, \ldots, P_K$, approximate it by set of K another polygonal curves $Q_1, Q_2, \ldots, Q_K$ with a given total number of segments M.

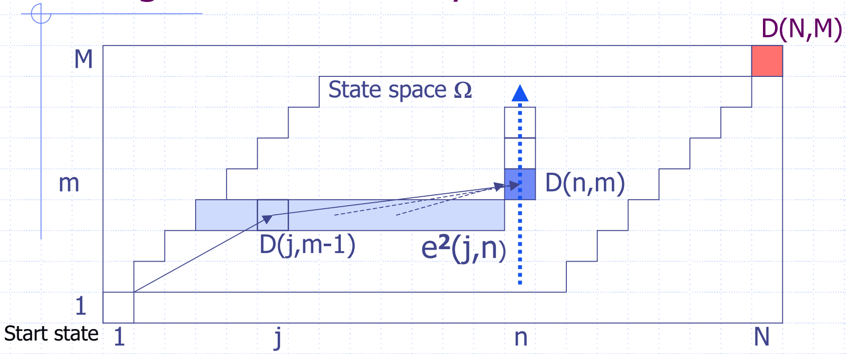
Multi-object *min-ε problem* (cont'd)

Given K polygonal curves $P_1, P_2, \ldots, P_K$, approximate it by set of K another polygonal curves $Q_1, Q_2, \ldots, Q_K$ with a given total number of segments M so that the total approximation error with measure L_2 is minimized.

$$E(P_1,\ldots,P_K,M) = \min_{\{M_k\}} \min_{\{q_m\}} \sum_{k=1}^{K} \sum_{m=1}^{M_k-1} e^2\left(q_{k,m}, q_{k,m+1}\right)$$

subject to: $\sum_{k=1}^{K} M_k \le M$

DP algorihm for *min-ε problem*



Dynamic programming algorithm:

$$D(n,m) = \min \{ D(j,m-1) + e^2(j,n) \mid m \le j < n \}$$

$\forall (n,m) \in \Omega$

Iterative reduced search for min-ε problem

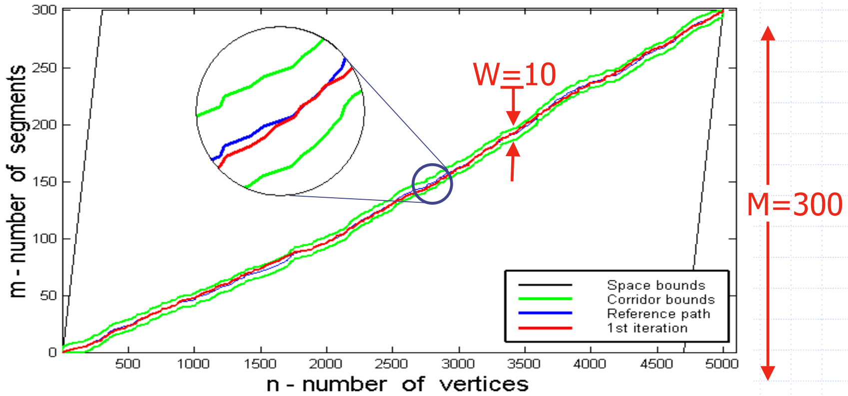
Step 1: Find preliminary approximation with any fast heuristic algorithm.

Step 2: Construct bounding corridor in state space along the reference path

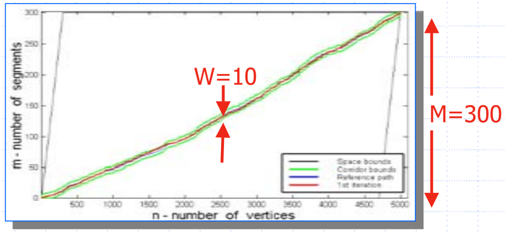
Step 3: Perform DP search in the bounding corridor

Use the output as a reference solution and repeat the search

Iterative reduced search DP algorithm



Iterative reduced search DP algorithm



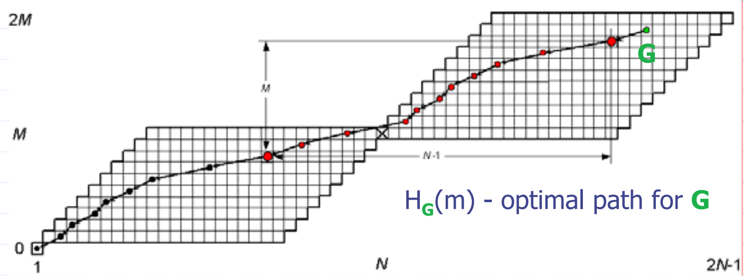
Complexity: $O(W^2N^2/M) = O(N)$-$O(N^2)$

Speed-up: $(W/M)^2$

Example: $(W/M)^2 = (10/300)^2 = 1/900$

Fidelity: up to 100% for iterative search

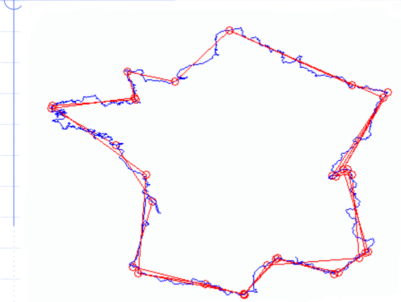
Min- ε problem for closed curves



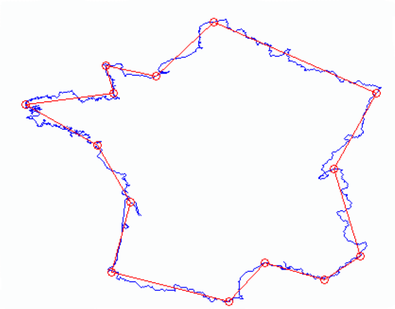
Conjugate states: $H_G(m-M) = H_G(m) - (N-1)$;

$n_{start} = \arg\min_{M \le m \le 2M} \{D(H_G(m),m) - D(H_G(m-M), m-M)\} - (N-1)$;

Min-ε problem for closed curves: results



Heuristic algorithm:
Fidelity F=88-100%
T=100 s



Proposed algorithm:
Fidelity F=100%
T=10.4 s.

Iterative reduced search for multi-object min-ε problem

Step 1: Find preliminary approximation of every object for an initial number of segments.

Step 2: Iterate the following:

a) Apply multiple-goal reduced search DP to define the Rate-Distortion functions.

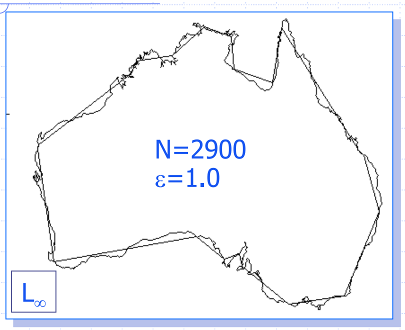
b) Solve the optimal allocation of the segments number among the objects using the Rate-Distortion functions.

Min-# solution as shortest path in digraph

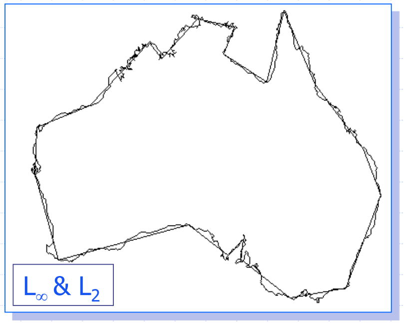
Feasiblity graph $G_\varepsilon(P)$ for ε=10 with L_∞ measure.

Min-# solution as shortest path in the digraph $G_\varepsilon(P)$.

Joint using of L_∞ & L_2 error measures



A shortest path in digraph $G_\varepsilon(P)$ for ε=1.0: E_2=629, d_{max}=1.0; T=4.5 s.



After optimization with reduced search: E_2=298, d_{max}=1.08; T=5.0 s.

Main results

1. Iterative Reduced Search algorithm, including:
   a. Algorithm for min-ε problem
   b. Algorithm for multiple-object min-ε problem
   c. Algorithm for min-# problem with L_∞ & L_2 error measures
2. Approximation algorithm for closed curves with state space analysis

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URL: http://cs.joensuu.fi/~koles/approximation/Ch3_0.html