Differential cryptanalysis of the quasigroup cipher

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- Motivation
- Definition of the encryption method
- Differential cryptanalysis
- Results
- Conclusions and Future Work

Quasigroup encryption

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- A *quasigroup* is a groupoid with unique left and right inverses.
- A quasigroup can be characterised with a $Latin\ square$ that is an $n * n$ matrix where each row and column is a permutation of elements of a set
- The encryption primitive e_l on sequence $x_1x_2 \ldots x_n$ is defined as $e_l(x_1x_2 \ldots x_n) = y_1y_2 \ldots y_n$ where

$$\begin{cases} y_1 = l * x_1, \\ y_{i+1} = y_i * x_{i+1} (i = 1, \ldots n - 1) \end{cases}$$

Encryption cont.

$$\begin{array}{l} \ \ a_1a_2a_3a_4a_5 \ldots \\ \mathrm{l}\ b_1b_2b_3b_4b_5 \ldots \\ \mathrm{l}\ c_1c_2c_3c_4c_5 \ldots \\ \ldots \end{array}$$

Decryption

- Decryption $d_l : A^+ \rightarrow A^+$ is defined as $d_l(y_1y_2 \ldots y_n) = x_1x_2 \ldots x_n$,where

$$\begin{cases} x_1 = l \backslash y_1, \\ x_{i+1} = y_i \backslash y_{i+1} (i = 1, \ldots n - 1) \end{cases}$$

Differential cryptanalysis on a Feistel cipher

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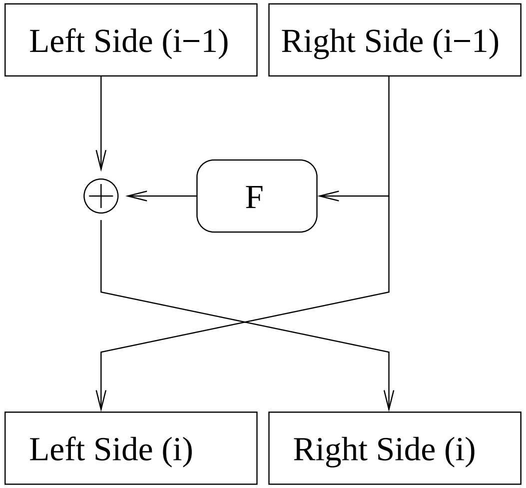
- A large amount of ciphertext - plaintext pairs is used

The Feistel structure



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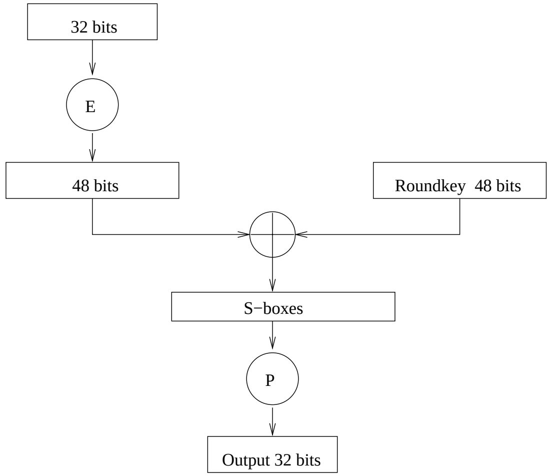

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- Output difference of the component $\Delta Z = (Y_1 \oplus K) \oplus (Y_2 \oplus K))$
- $\Delta Z = Y_1 \oplus Y_2$, since $(Y_1 \oplus K) \oplus (Y_2 \oplus K)$ = $Y_1 \oplus Y_2 \oplus K \oplus K$.

Differential analysis of a quasigroup

```
for (a_1 := 0 ... Quasigroupsize)
  for (a_2 := 0 ... Quasigroupsize)
    for (leader := 0 ... Quasigroupsize)
      c_1 := e_transformation(leader,a_1)
      c_2 := e_transformation(leader,a_2)
      input_xor := a_1 ⊕ a_2
      output_xor := c_1 ⊕ c_2
      distributions[input_xor][output_xor]++
    endfor
  endfor
endfor
```

3	12	6	14	8	9	13	11	15	4	1	5	10	7	0	2
4	10	14	9	0	12	7	5	11	8	3	15	1	6	2	13
12	9	1	3	14	11	2	8	13	5	6	0	7	15	10	4
15	5	10	11	7	14	4	13	3	0	2	1	12	8	9	6
0	11	3	10	13	5	8	14	1	15	12	9	6	2	4	7
10	1	8	12	11	0	5	3	9	13	4	7	2	14	6	15
6	4	15	13	1	7	14	9	8	10	5	2	11	3	12	0
5	15	13	2	9	10	1	12	0	6	7	14	4	11	3	8
13	6	7	1	2	8	9	10	14	3	15	4	0	5	11	12
14	7	11	4	3	2	15	0	12	9	8	6	5	10	13	1
9	13	2	0	15	4	10	7	6	12	11	3	14	1	8	5
8	14	5	15	6	1	0	4	10	2	9	11	13	12	7	3
11	2	9	6	5	13	12	15	4	7	10	8	3	0	1	14
1	3	0	8	10	15	6	2	7	14	13	12	9	4	5	11
7	0	12	5	4	6	3	1	2	11	14	10	8	13	15	9
2	8	4	7	12	3	11	6	5	1	0	13	15	9	14	10

An example quasigroup of order 16

$I \backslash O$	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0000	256	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0001	0	10	24	22	24	14	20	14	18	20	14	16	10	16	10	24
0010	0	20	20	26	10	28	10	22	12	12	14	20	12	22	14	14
0011	0	26	14	16	12	22	6	12	28	16	24	24	18	18	10	10
0100	0	14	10	18	20	16	20	22	20	12	14	24	10	12	30	14
0101	0	18	20	20	18	14	18	16	10	18	18	24	12	18	16	16
0110	0	20	14	16	20	22	10	18	26	18	14	12	8	14	24	20
0111	0	8	14	18	24	16	24	16	14	24	22	16	10	12	16	22
1000	0	16	26	22	14	18	12	12	14	18	14	18	28	20	12	12
1001	0	16	16	20	8	20	16	16	12	12	20	12	24	12	24	28
1010	0	24	28	8	18	18	18	22	8	20	16	8	14	18	14	22
1011	0	24	20	6	10	20	14	14	16	22	22	18	18	18	20	14
1100	0	12	12	18	18	10	20	18	14	14	12	26	26	34	14	8
1101	0	10	20	12	22	16	22	18	20	18	20	24	10	12	14	18
1110	0	14	6	20	20	8	22	18	18	18	20	8	34	12	20	18
1111	0	24	12	14	18	14	24	18	26	14	12	6	22	18	18	16

An example difference distribution table

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0
2	3	4	5	6	7	8	9	10	11	12	13	14	15	0	1
3	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2
4	5	6	7	8	9	10	11	12	13	14	15	0	1	2	3
5	6	7	8	9	10	11	12	13	14	15	0	1	2	3	4
6	7	8	9	10	11	12	13	14	15	0	1	2	3	4	5
7	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6
8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7
9	10	11	12	13	14	15	0	1	2	3	4	5	6	7	8
10	11	12	13	14	15	0	1	2	3	4	5	6	7	8	9
11	12	13	14	15	0	1	2	3	4	5	6	7	8	9	10
12	13	14	15	0	1	2	3	4	5	6	7	8	9	10	11
13	14	15	0	1	2	3	4	5	6	7	8	9	10	11	12
14	15	0	1	2	3	4	5	6	7	8	9	10	11	12	13
15	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

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0000	256	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0001	0	128	0	64	0	0	0	32	0	0	0	0	0	0	0	32
0010	0	0	128	0	0	0	64	0	0	0	0	0	0	0	64	0
0011	0	64	0	64	0	32	0	32	0	0	0	0	0	32	0	32
0100	0	0	0	0	128	0	0	0	0	0	0	0	128	0	0	0
0101	0	0	0	32	0	64	0	32	0	0	0	32	0	64	0	32
0110	0	0	64	0	0	0	64	0	0	0	64	0	0	0	64	0
0111	0	32	0	32	0	32	0	32	0	32	0	32	0	32	0	32
1000	0	0	0	0	0	0	0	0	256	0	0	0	0	0	0	0
1001	0	0	0	0	0	0	0	32	0	128	0	64	0	0	0	32
1010	0	0	0	0	0	0	64	0	0	0	128	0	0	0	64	0
1011	0	0	0	0	0	32	0	32	0	64	0	64	0	32	0	32
1100	0	0	0	0	128	0	0	0	0	0	0	0	128	0	0	0
1101	0	0	0	32	0	64	0	32	0	0	0	32	0	64	0	32
1110	0	0	64	0	0	0	64	0	0	0	64	0	0	0	64	0
1111	0	32	0	32	0	32	0	32	0	32	0	32	0	32	0	32

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Brute force attack

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- $|Q|$ different leaders
- We need in average $\frac{(|Q|*k)^n}{2}$ trials
- k is the number of different operations used

$\|Q\|$	5	10	20	30	40
4	$1.53 * 2^{21}$	$1.16 * 2^{43}$	$1.36 * 2^{86}$	$1.58 * 2^{129}$	$1.84 * 2^{172}$
8	$1.53 * 2^{26}$	$1.16 * 2^{53}$	$1.36 * 2^{106}$	$1.58 * 2^{159}$	$1.84 * 2^{212}$
16	$1.53 * 2^{31}$	$1.16 * 2^{63}$	$1.36 * 2^{126}$	$1.58 * 2^{189}$	$1.84 * 2^{252}$
32	$1.53 * 2^{36}$	$1.16 * 2^{73}$	$1.36 * 2^{146}$	$1.58 * 2^{219}$	$1.84 * 2^{292}$
64	$1.53 * 2^{41}$	$1.16 * 2^{83}$	$1.36 * 2^{166}$	$1.58 * 2^{249}$	$1.84 * 2^{332}$
128	$1.53 * 2^{46}$	$1.16 * 2^{93}$	$1.36 * 2^{186}$	$1.58 * 2^{279}$	$1.84 * 2^{372}$
256	$1.53 * 2^{51}$	$1.16 * 2^{103}$	$1.36 * 2^{206}$	$1.58 * 2^{309}$	$1.84 * 2^{412}$

Amount of tries needed for a quasigoup of size $|Q|, k = 5$.

In colums are values calculated for 5, 10, 20, 30 and 40 iterations of the cipher.

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- In a brute force attack we need to
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- Find out the leader(s) used to encrypt the message
- Find out the order in which the quasigroups were used to encrypt the message

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- There is no (known) formula for deciding the amount of latin squares of certain order.
- Experiments show that there are 576 latin squares of order 4, more than 10^{90} of order 16.
- For simplicity we assume that we know how many encryptions has been done (pherhaps we can use timing attack to find this out).

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- A brute force attack after this consists of trying all the possible quasigroups with all the possible leaders, succeeding within average of half the the possibities tried.

- Unfortunately we do not know how many possibilities there are for quasigroups of order higher than ?

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$$\begin{array}{cccc} \hline x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \\ \hline \end{array}$$

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- where for example x_{11} = x_{22} = x_{34} = x_{43} = 01

Finding bit difference patterns (1 round)

After 1 round of encryption we can find bit differences as follows:

$$a_1 a_2 a_3 a_4 a_5 a_6 \dots$$

$$b_1 b_2 b_3 b_4 b_5 b_6 b_7 \dots$$

After 2 rounds of encryption we can find bit differences as follows:

$$a_1 a_2 a_3 a_4 a_5 a_6 \dots$$

$$b_1 b_2 b_3 b_4 b_5 b_6 b_7 \dots$$

$$c_1 c_2 c_3 c_4 c_5 c_6 c_7 \dots$$

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01	0	8	8	0
10	0	8	8	0
11	0	0	0	16

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$I\backslash O$	00	01	10	11
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10	0	8	8	0
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- We can now construct the quasigroup structure using the difference table.

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00	x_{12}	x_{13}	11
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11	x_{42}	x_{43}	00

Finding the quasigroup

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00	x_{12}	x_{13}	11
01	x_{22}	x_{23}	10
10	x_{32}	x_{33}	01
11	x_{42}	x_{43}	00

- For difference 01 we have two charasteristics with equal probability, namely 01 → 01 and 01 → 10.

00	01	x_{13}	11
01	x_{22}	x_{23}	10
10	x_{32}	x_{33}	01
11	x_{42}	x_{43}	00

00	10	x_{13}	11
01	x_{22}	x_{23}	10
10	x_{32}	x_{33}	01
11	x_{42}	x_{43}	00

Finding the quasigroup

- For determining the value of x_{22} we have to do the same ending up with four possible tables:

00	01	10	11
01	00	11	10
10	x_{32}	x_{33}	01
11	x_{42}	x_{43}	00

00	10	01	11
01	00	11	10
10	x_{32}	x_{33}	01
11	x_{42}	x_{43}	00

00	01	10	11
01	11	00	10
10	x_{32}	x_{33}	01
11	x_{42}	x_{43}	00

00	10	01	11
01	11	00	10
10	x_{32}	x_{33}	01
11	x_{42}	x_{43}	00

Finding the quasigroup

- For x_{32} we have two choices
- This means eight possible structures four of which would violate the definition of latin squares

00	01	10	11
01	00	11	10
10	11	00	01
11	10	01	00

00	10	01	11
01	00	11	10
10	11	00	01
11	01	10	00

00	01	10	11
01	11	00	10
10	00	11	01
11	10	01	00

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- The task of finding the latin square from difference distributions comes more difficult as the order increases.

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Attack with known structure

- Knowing the structure of the latin square reduces brute force complexity to n!
- For example for order 4 this is 24 while amount of latin squares of order 4 is 576.
- The task of finding the latin square from difference distributions comes more difficult as the order increases.
- Some cases are "simple" and some can be impossible

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Using several quasigroups

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- Using two latin squares with distribution tables of, for example, form

	00	01	10	11
00	16	0	0	0
01	0	8	0	8
10	0	0	16	8
11	0	8	0	8

	00	01	10	11
00	16	0	0	0
00	0	16	0	0
00	0	0	8	8
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00	12	0	0	0
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11	0	1	2	3

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- will give the most uniform distribution, such as

00	12	0	0	0
01	0	3	2	2
10	0	3	4	5
11	0	1	2	3

- which reveals nothing about the structures of the groups.

Conclusions

- In some cases it is possible to gain considerable advantage with differential analysis compared to straight brute force attack
- It is useful to consider a difference distribution of a quasigroup before using it
- If small group is used, one should use more than one group
- These groups should be selected so that combined they produce difference distribution that has no charasteristics with probability 1.

Future work

- One could generate a general algorithm for finding the quasigroup based on the difference distribution
- What happens if one uses different, but isomorphic quasigroups (ie. quasigroups with same structure) for encryption and decryption?

Thank you

Questions?