Weak Regenerative Simulation: Some Numerical Examples for Queues and Queuing Networks

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Abstract

In the paper we apply the so–called regenerative approach to constructing confidence intervals for the steady–state average of the basic queuing processes. A few numerical results show that the extended (weak) regenerative approach leads sometimes to reduction of simulation time.

1 Introduction

Many queuing processes are too complex to be investigated by pure analytical methods or by classical statistical procedures only. These procedures deal with independent and identically distributed (i.i.d.) random variables. At the same time real–life stochastic processes have complex correlation structure (strong dependence between values of the process). For instance, consider a waiting time process $W = \{W_n\}$ in a general system $GI/G/1$, where $\{W_n\}$ is the waiting time of n–th customer in the queue. Let us denote $\{t_n\}$—arrival instants, $\{\tau_n = t_{n+1} - t_n\}$—i.i.d.

interarrival times, $\{S_n\}$—i.i.d. service times. The following relation is well–known [1]:

$$W_{n+1} = \max(0, W_n + S_n - \tau_n),\ n \geq 0.$$

Variables $\{W_n\}$ are strongly correlated (during busy periods) and this dependence causes much problems during simulation:

- is corresponding estimate consistent and unbiased?
- how to construct the confidence intervals?
- how to choose the starting and finishing points of the simulation procedure (to remove the influence of transient period)?

The so–called regenerative approach was developed by W. L. Smith in the middle of 50th [10] and nowadays it found wide applications, in particular, to simulating of queuing networks. This approach is based on the fact, that each time the network becomes empty, it starts a new regeneration cycle and all cycles are i.i.d. It is then possible to obtain reliable estimates of cycle characteristics using the classical statistical properties of sequences of i.i.d. random variables. This classical approach is well–known and we will not discuss it in this paper.

2 Classic regeneration

To extend the possibilities of the regenerative approach we consider the so–called weakly regenerative processes now. [1, 7, 5]. This definition can be obtained from the definition of strong regeneration by the replacing the condition (ii) by the following weaker condition:

(ii) $\Theta_{\beta_k}(X, \beta)$ does not depend on $(\beta_0, \ldots, \beta_k)$, for all $k \geq 0$

In other words, weak regeneration admits dependence between r.c's but their lengths $\{\alpha_n\}$ stay i.i.d. Let us get back to our examples.

Example 1. Consider m–server node $GI/G/m$. We introduce the following event (depending on index n): $\mathcal{E} = \Big\{$ *there are m successive customers $n, ...n+m-1$: each finds an empty new server* $\Big\}$. Then on the event $\mathcal{E}$ the discrete time point $n+m-1$ is weak r.p. for any (discrete–time) queuing process.

Example 2. Suppose that in an open network the following event holds: {there is a customer which crosses the network "without contacts" with other customers}. Then a weak r.p. is the next arrival point after this "crossing" (see [5,7] for more detail).

When an arbitrary dependence between the r.c.'s is assumed, we may apply the following version of the central limit theorem [2] (see also [3]):

If $\{Z_i\}$ are m–dependent identically distributed random variables with zero expectation and $VarZ_1^2 < \infty$, then

$$\frac{\sum_{i=1}^{k} Z_i}{\sqrt{\mathrm{Var}\sum_{i=1}^{k} Z_i}} \Rightarrow N(0,1), \text{ as } k \to \infty.$$

We denote $Z_i = Y_i - \alpha_i EX$ ($EZ_i = 0$). To estimate $Var\{\sum_{i=1}^{k} Z_i\}$, we can apply Cauchy–Bunyakovsky inequality. It gives us the following result:

$$D(\sum_{i=1}^{n} Z_i) = \sum_{i=1}^{n} DZ_i + 2\sum_{i=1}^{n-1} cov(Z_i, Z_{i+1}) =$$

$$nDZ_1 + 2(n-1)cov(Z_1, Z_2) =$$

$$nDZ_1 + 2(n-1)E(Z_1 \cdot Z_2) \le nDZ_1 + 2(n-1)\sqrt{E(Z_1)^2 E(Z_2)^2} =$$

$$nDZ_1 + 2(n-1)E(Z_1)^2 = (3n-2)DZ_1$$

This allows us to obtain the following $100(1-2\gamma)\%$ confidence interval for the characteristic $E\{f(X^*)\}$ (see [6])

$$I(n) = \left[\hat{r}(n) - \frac{z_{1-\gamma} s(n)\sqrt{3n-2}}{n\bar{\alpha}_n}, \hat{r}(n) + \frac{z_{1-\gamma} s(n)\sqrt{3n-2}}{n\bar{\alpha}_n}\right]$$

Hence, one can construct confidence intervals for various steady–state characteristics of the network process $\{X\}$ (waiting time, sojourn time, queue–size) using weak or strong regeneration.

3 Numerical results

Now we consider some numerical results of the queuing process simulation, where the number of weak r.p.'s appear essentially more frequent than the strong r.p.'s. It should be noted that in each experiment we use 1000 customers (observation series) and each series is repeated 30 times.

Example 1. Let us consider the GI/G/3 system with the service time distribution (first column contains values of random variable and second one—the corresponding probabilities):

$$s = \begin{cases} 2, & 0,2; \\ 0,6, & 0,1; \\ 0,7, & 0,7. \end{cases}$$

and interarrival time distribution

$$\tau = \begin{cases} 0,28, & 0,04; \\ 1, & 0,03; \\ 0,57, & 0,93. \end{cases}$$

The following results are obtained: 10 strong r.p.'s and 600 weak r.p.'s.

Example 2. In GI/G/3 system with service–time distribution being

$$s = \begin{cases} 6, & 0,9; \\ 10, & 0,09; \\ 100, & 0,01. \end{cases}$$

and interarrival time distribution

$$\tau = \begin{cases} 3, & 1/3; \\ 5, & 2/3; \end{cases}$$

there are 1 strong r.p. and 740 weak r.p.'s.

Example 3. In GI/G/5 system with service time distribution

$$s = \begin{cases} 6, & 0,9; \\ 20, & 0,09; \\ 200, & 0,01. \end{cases}$$

and interarrival time distribution

$$\tau = \begin{cases} 2, & 1/3; \\ 3, & 2/3; \end{cases}$$

there are 1 strong r.p. and 50 weak r.p.'s.

Example 4. In GI/G/10 system with service time distribution

$$s = \begin{cases} 9.9, & 0,9; \\ 100, & 0,09; \\ 200, & 0,01. \end{cases}$$

and interarrival time distribution

$$\tau = \begin{cases} 2, & 1/3; \\ 3, & 2/3; \end{cases}$$

there are 1 strong r.p. and 50 weak r.p.'s.

Example 5. For the tandem network with 4 nodes, service time distribution

$$s = \begin{cases} U[1,3], & 0,95; \\ 12, & 0,04; \\ 15, & 0,01. \end{cases}$$

and interarrival time distribution $\tau \sim U[1,7]$

we obtain 2 strong r.p.'s and 98 weak r.p.'s.

4 Conclusion

First of all, we give Figure 1 and Figure 2 which illustrate the numerical results related to the sojourn time in tandem network considered in example 5. More exactly, the pictures Figure 1 and Figure 2 show the dependence between width of confidence intervals (for the sample mean of the sojourn time), the number of r.c.'s (N) and number of observations (n) for the strong and weak r.p.'s respectively.

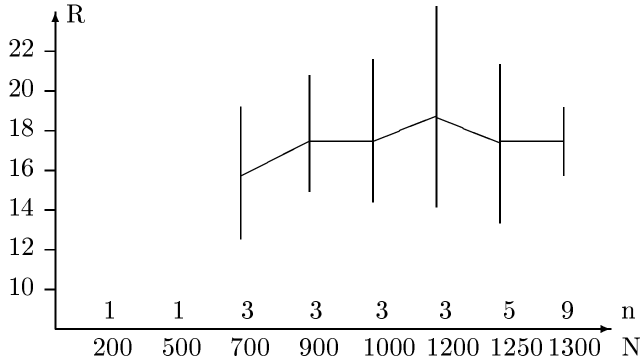

Figure 1. Estimation by using strong r.p.'s

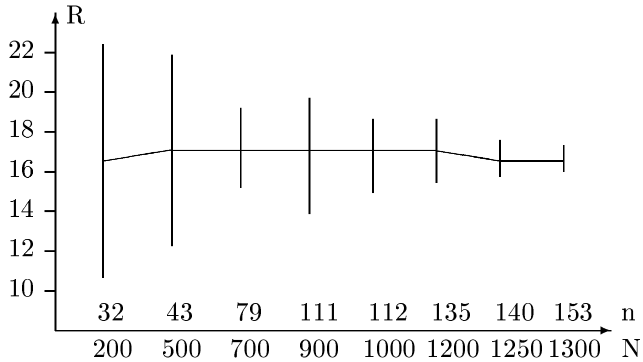

Figure 2. Estimation by using weak r.p.'s

We note that if the regenerative structure of the system makes the appearance of strong r.p.'s impossible, then the observation of the weak r.p.'s is the only way to construct the confidence interval.

At the same time, the queuing systems have often both strong r.p.'s and weak r.p.'s. In our examples the number of weak r.p.'s is essentially greater than the number of strong r.p.'s. This property of the queuing process can decrease simulation time in the case when shorter regeneration cycles imply variance reduction of the corresponding statistical estimates. (There are a number of settings where shorter cycles are more effective in the simulation, see, for example, [3].)

In this connection it is very important to describe a class of distributions of interarrival and service times where weak regeneration has advantage mentioned above. There is a serious foundation to suppose that the so–called heavy–tailed distribution (the distribution with infinite variance) has the desired property. We emphasize that applications of the regenerative approach in network simulation is yet restricted by the strong regeneration only (for example, [9]). This is an another reason why the development of weak regeneration approach seems very perspective and important in the simulation aspect.

References

[1] *S. Asmussen* Applied probability and queues, Wiley, 1987.

[2] *I. Ibragimov* A note on CLT for dependent random variables Theory probability and applications, 1975, v. 20, No 1, 134–140.

[3] *P. Glynn* Some topics in regenerative stately–state simulation, Acta Appl. Math., 1994, v.34, No 1–2, 225–236.

[4] *S. Janson* Annals of Probability, Renewal theory for m–dependent variables. 1983, v. 11, No 3, 558–568.

[5] *S. Foss, V. Kalashnikov* Regeneration and renovation in queues, Queuing Systems, 1991, v.8, 211–224.

[6] *E. Morozov* Wide sense regenerative processes with applications to multi–channel queues and networks, Acta Appl.Math., 1994, v. 34, 189–212.

[7] *E. Morozov and S. Sigovtsev* Simulation of queuing processes based on weak regeneration, Journal of Mathematical Sciences, 1998, v.89, No 5, 1517–1523.

[8] *E. Morozov* The tightness in the ergodic analysis of regenerative queuing processes, Queuing Systems, 1997, v. 27, 179–203.

[9] *G. S. Shedler* Regeneration and networks of queues, Springer–Verlag, 1986.

[10] *W. L. Smith* Regenerative stochastic processes, Proc. Royal Soc., A, 1955, v. 232, 6-31.